Physics

1. Yes, at midway between two equal and opposite charges, electric potential is zero but electric field exists **(1 mark)**

2. The temperature at which a substance losses its ferromagnetic behaviour and becomes paramagnetic **(1 mark)**

3. Magnetic Lorentz force is the force acting on a charge in a magnetic field. $\left(\frac{1}{2}\textbf{ mark}\right)$

$\vec{F} = q(\vec{v} \times \vec{B})$ $\left(\frac{1}{2}\textbf{ mark}\right)$

4. This is done to prevent huge loss of energy due to eddy currents **(1 mark)**

5. AGC means automatic gain control. $\left(\frac{1}{2}\textbf{ mark}\right)$

It is used in a receiver to keep the strength of the received signal constant. $\left(\frac{1}{2}\textbf{ mark}\right)$

6. Initial force is $F_1 = k\dfrac{q_1q_2}{r^2}$ **(1 mark)**

If distance as well as the charge on each sphere is doubled, then

the new force, $F_2 = k\dfrac{2q_1 \times 2q_2}{(2r)^2} = k\dfrac{4q_1q_2}{4r^2} = k\dfrac{q_1q_2}{r^2} = F_1$ itself. **(1 mark)**

i.e. there is no change in the force.

7. 

Let $q_1 = 5 \times 10^{-5}$ C and $q_2 = -2 \times 10^{-5}$ C be at 0.1 m apart.(10 cm = 0.1 m). Also let at a distance x from q_1 the potential be zero. Then,

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1}{x} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{[0.1 - x]} = 0$$ **(1 mark)**

$$\frac{5\times10^{-5}}{x}+\frac{-2\times10^{-5}}{0.1-x}=0$$

$$\frac{5}{x}=\frac{2}{0.1-x}$$

$$5(0.1-x)=2x$$

$$x=0.071\text{m}=7.1\text{ cm}$$ **(1 mark)**

The potential is zero at a point 7.1cm from the charge 5×10^{-5} C

8. Resistivity of a material is defined as the resistance offered by 1 m length of the material having a cross-sectional area of 1 m^2. **(1 mark)**
For metals, resistivity increases when temperature increases while for semiconductors, electrolytes, etc. the resistivity decreases with temperature. **(1 mark)**

9. The emf induced in the coil is given by

$$e=-\frac{d\phi}{dt}$$ $\left(\frac{1}{2}\textbf{ mark}\right)$

but ϕ = Li where L is the self inductance

$$e=-L\frac{di}{dt}$$ $\left(\frac{1}{2}\textbf{ mark}\right)$

The work done in maintaining a current i for time dt is given by

$$dw=-e.dq$$

$$=-eidt$$

$$L\frac{di}{dt}\times idt$$

$$=Lidi$$ $\left(\frac{1}{2}\textbf{ mark}\right)$

Total work done in maintaining the current i0 is

$$W=\int_o^{i_o}Lidi=\frac{Li_o^2}{2}$$

this energy is stored in the coil. $\left(\frac{1}{2}\textbf{ mark}\right)$

10. When two thin lens of focal length F_1 and F_2 are in contact with each other, their combined focal length F is given by

$$\frac{1}{F} = \frac{1}{F_1} + \frac{1}{F_2}$$ $\left(\frac{1}{2}\text{ mark}\right)$

Now

F_1 = +30 cm (convex lens)

F_2 = –20 cm (concave lens) $\left(\frac{1}{2}\text{ mark}\right)$

$$\frac{1}{F} = \frac{1}{30} - \frac{1}{20} = \frac{2-3}{60}$$

$$\frac{1}{F} = -\frac{1}{60}$$

F = –60 cm

Or

Given A = 6°, $\delta = 3°$, $\mu = ?$

Using $\delta = (\mu - 1)\ A$, $\left(\frac{1}{2}\text{ mark}\right)$

$$3° = (\mu - 1)\ 6°$$

$$(\mu - 1) = \frac{3°}{6°} = 0.5$$ $\left(\frac{1}{2}\text{ mark}\right)$

or $\mu = 1.5$

11. Following are some properties of cathode rays.

(i) Cathode rays travel in the straight line and cast a sharp shadow of the object kept in its path.

(ii) Cathode rays are emitted in a direction normal to the surface of the cathode and it is independent of the position of the anode.

(iii) These rays are found to possess kinetic energy and can exert mechanical pressure
(iv) Cathode rays produce heat.
(v) They are reflected by electric and magnetic fields
(vi) Cathode rays are independent of the nature of the gas or the material used in the discharge tube. Therefore, the value of the specific charge for the cathode rays is universal constant and is given by $\frac{e}{m} = 1.7592 \times 10^{11}\, C kg^{-1}$. This also proves that they are electrons.
(Write any four. Other valid properties will be awarded full marks.)

12. V = 60 V
m = 9 × 10^{-31} kg
v = 1. 6 ×10^{-19} C

h = 6.6 × 10^{-34} Js $\left(\frac{1}{2}\textbf{ mark}\right)$

$$\gamma = \frac{h}{\sqrt{2\,emV}}$$ $\left(\frac{1}{2}\textbf{ mark}\right)$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 1.6 \times 10^{-19} \times 10^{-31} \times 60}} = 1.58 \times 10^{-10} m$$ **(1mark)**

13. If the dielectric constant of the material is K, then

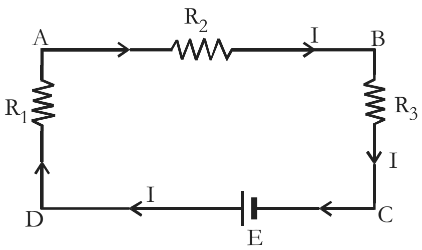
(i) the potential difference decreases to $\frac{1}{K}$ times the initial value. **(1 mark)**

(ii) capacity increases to K times the initial value. **(1 mark)**

(iii) energy decreases to $\frac{1}{K}$ times the initial value **(1 mark)**

14. Resistors in series

Let three resistances R_1 , R_2 and R_3 be in series



$\left(\frac{1}{2}\text{ mark}\right)$

The current is same in each resistor. V_1, V_2 and V_3 are the values of potential difference across R_1, R_2 and R_3 respectively, then

$V_1 = IR_1, V_2 = IR_2$ and $V_3 = IR_3$

Also, $V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3$ $\left(\frac{1}{2}\text{ mark}\right)$

If the resistors are replaced by their equivalent resistance R, we could write

$V = IR$ …(iii) $\left(\frac{1}{2}\text{ mark}\right)$

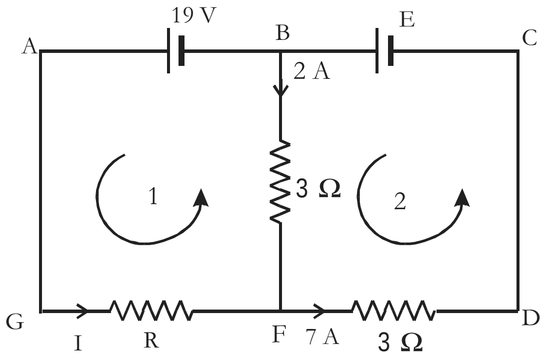
or $I(R_1+R_2+R_3) = IR$ …(iv) $\left(\frac{1}{2}\text{ mark}\right)$

$R_1 + R_2 + R_3 = R$ $\left(\frac{1}{2}\text{ mark}\right)$

the equivalent resistance of any number of resistors in series

equals the sum of their individual resistances. $\left(\frac{1}{2}\text{ mark}\right)$

15.



From loop 2, $\Sigma E = \Sigma IR$

$-E = 6 + 21$ ... (i)

$E = -27$ V **(1 mark)**

From loop 1, $-19 = IR - 6$

$IR = -13$… (ii)

At F, from Kirchhoff's current law, $2 + I = 7$

$I = 5$ A **(1 mark)**

Using (ii)

$5R = -13$

$R = \frac{-13}{5} = 2.6\ \Omega$ numerically **(1 mark)**

Or

(i) In a voltaic cell the iron and lead impurities at the surface of zinc, in contact with sulphuric acid form tiny local voltaic cells. As a result, the zinc plate goes on dissolving even when the cell is not sending any external current. **(1 mark)**

(ii) Given,

Mass of copper to be deposited (M) = 0.254 kg

Current (I) = 100 A

Number of equivalents in 0.254 kg or 254 g of Cu

$= \frac{254 \times 2}{63.5} = 8$ **$\left(\frac{1}{2}\text{ mark}\right)$**

Charge (q) = 8 F = 8 × 96500 C **$\left(\frac{1}{2}\text{ mark}\right)$**

$I = \frac{q}{t}$ **$\left(\frac{1}{2}\text{ mark}\right)$**

or $t = \frac{q}{I} = \frac{8 \times 96500}{100} = 7720\text{ s}$ **$\left(\frac{1}{2}\text{ mark}\right)$**

16. (i) $Bv = B_H$

$\tan\delta = \frac{B_V}{B_H} = 1$

$\delta = \tan^{-1}(1) = 45°$ **$\left(1\frac{1}{2}\text{ marks}\right)$**

(ii) $\cos\delta = \frac{B_H}{B} = \frac{1}{\sqrt{2}}$

or $\delta = 45°$. **$\left(1\frac{1}{2}\text{ marks}\right)$**

17. The torque acting on a magnetic dipole of moment M placed in a uniform field of strength B will be

$\tau = MB\sin\theta$ **$\left(\frac{1}{2}\text{ mark}\right)$**

The work in rotating it through an angle $d\theta$ wil be,

$dw = \tau d\theta = MB\sin\theta d\theta$ **$\left(\frac{1}{2}\text{ mark}\right)$**

the total work done to rotate the dipole from initial position

$\theta = \theta_1$ to $\theta = \theta_2$,

$$W = \int_{\theta_1}^{\theta_2} MB\sin\theta d\theta = MB\int_{\theta_1}^{\theta_2} \sin\theta d\theta = MB[-\cos\theta]_{\theta_1}^{\theta_2}$$

$= -MB(\cos\theta_2 - \cos\theta_1)$ **$\left(\frac{1}{2}\text{ mark}\right)$**

The work done is stored as potential energy in it.

$U = -MB(\cos\theta_2 - \cos\theta_1)$ **$\left(\frac{1}{2}\text{ mark}\right)$**

The PE of the dipole is zero when $\theta = 90°$. Then PE in any position can be obtained by setting $\theta_1 = 90°$ and $\theta_2 = \theta$ **$\left(\frac{1}{2}\text{ mark}\right)$**

With these, $U = -MB(\cos\theta - \cos 90°)$

$= -MB\cos\theta$

$= \vec{M}\cdot\vec{B}$ **$\left(\frac{1}{2}\text{ mark}\right)$**

18. Intensity of the electromagnetic wave is defined as the energy crossing per second per unit area perpendicular to the direction of propagation of the electromagnetic wave. **(1 mark)**

In terms of maximum electric field,

$$I = \frac{1}{2}\varepsilon_0 E_0{}^2 C = \varepsilon_0 E^2{}_{rms} C$$ **(1 mark)**

In terms of maximum magnetic field,

$$I = \frac{1}{2}\frac{B^2{}_0}{\mu_0} C = \frac{1}{\mu_0} B^2{}_{rms} C$$ **(1 mark)**

19. When light passes from a denser medium to a rarer medium and the angle of incidence exceeds a certain value called critical angle , the ray cannot get out of the denser medium. Instead it gets reflected back into the denser medium itself. This phenomenon is known as total internal reflection. **(1 mark)**

The necessary and sufficient conditions for total internal reflection are,

(i) light should pass from a denser medium to a rarer medium **(1 mark)**

(ii) the angle of incidence should be greater than the critical angle. **(1 mark)**

20. (a) Two limitations of Thomson's model are:

(i) The large scattering of the α-particle, when bombarded on thin gold foil, could not be explained.

(ii) The origin of the spectrum could not be explained. **(1mark)**

(b) For the smallest orbit of hydrogen atom, n = 1. Radius is

$$r_1 = \frac{n^2 h^2}{4\pi^2 mKe^2}$$ **(1mark)**

Substituting the values of h, m, e and K, we get

$$r_1 = 5.29 \times 10^{11}\ \text{m} = 0.53\ \text{Å}$$ **(1 mark)**

21. The decay constant of a radioactive substance is the reciprocal of the time at the end of which, the number of parent atoms left in a radioactive sample is $\left(\frac{1}{e}\right)$ times the number of atoms in the original sample (N_0). **(1 mark)**

We know, $N = N_0 e^{-\lambda t}$ $\left(\frac{1}{2}\text{ mark}\right)$

Half life is the time taken by the sample to disintegrate into half of the initial amount. $\left(\frac{1}{2}\text{ mark}\right)$

At, t = T, $N = \frac{N_0}{2}$

Then, $\frac{N_0}{2} = N_0 e^{-\lambda T}$ **$\left(\frac{1}{2}\text{ mark}\right)$**

Solving, $T = \frac{0.693}{\lambda}$ **$\left(\frac{1}{2}\text{ mark}\right)$**

22. Doping is the process of adding suitable impurity atoms to an intrinsic semiconductor in order to increase its conductivity so as to become a part of the crystal structure. Trivalent impurities make a crystal rich in holes while pentavalent impurities make them rich in free electrons. **$\left(1\frac{1}{2}\text{ marks}\right)$**

(i) As electron concentration is increased, the impurity added is pentavalent. **$\left(\frac{1}{2}\text{ mark}\right)$**

(ii) $n_i = 2 \times 10^8\ m^{-3}$ $n_e = 3.5 \times 10^8\ m^{-3}$

$$n_h = \frac{n_i^2}{n_e} = \frac{(2\times10^8)^2}{3.5\times10^8} = 1.14 \times 10^8 m^{-3}$$ **$\left(\frac{1}{2}\text{ mark}\right)$**

(iii) Doping decreases the energy gap. **$\left(\frac{1}{2}\text{ mark}\right)$**

23. (i) Optical fibre communication system has very large bandwidth.

(ii) Optical fibres have very small diameters and are far smaller and lighter than coaxial cables.

(i) Optical fibres are insulators (electrically) and do not show earth loop or interface problems like metallic cables.

(ii) Optical fibres serve as dielectric wave-guides and therefore are free from electromagnetic and radio frequency interference.

(iii) The light from optical fibres do not radiate significantly. Hence optical fibre communication provides a high degree of signal security.

(iv) Transmission loss is very low in optical fibre communication.

$\left(\frac{1}{2}\times 6 = 3\text{ marks}\right)$

24. Given, h = 80 m and p = 800/km2

Coverage range,

$d = \sqrt{2hR} = \sqrt{2 \times 80 \times 6.4 \times 10^6} = 4 \times 8 \times 10^3 \text{ m} = 32 \text{ km}$ **(1 mark)**

Population covered, $P = p \times \pi d^2 = 800 \times 3.14 \times 32^2 = 25,72,288$

(1 mark)

If the coverage value is to be doubled, let h' be the new height, then, but

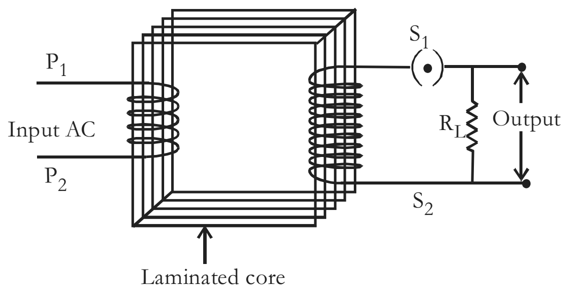
$d' = \sqrt{2h'R}$

$d' = 2d$

$\sqrt{2h'R} = 2\sqrt{2hR}$ **(1 mark)**

$h' = 4h = 4 \times 80 = 320$ m.

25. (i) Transformer works on the principle of mutual induction. If two coils are inductively coupled and current or magnetic flux linked with one is changed, then an induced emf is produced in the other. $\left(\frac{1}{2}\text{ mark}\right)$



(1 mark)

ii) $N_1 = 200$, $N_2 = 1000$, $V_2 = 1000V$, $\eta = 90\%$.

As $\eta = \frac{P_0}{P_1}$ $\left(\frac{1}{2}\text{ mark}\right)$

or $P_1 = \frac{P_0}{\eta} = \frac{9}{\frac{90}{100}} = 10 \text{ kW}$ $\left(\frac{1}{2}\textbf{ mark}\right)$

(a) $\frac{V_2}{V_1} = \frac{N_2}{N_1}$ $\left(\frac{1}{2}\textbf{ mark}\right)$

or $V_1 = \frac{N_1}{N_2} \times V_2 = \frac{200 \times 1000}{1000} = 200 \text{ V}$ $\left(\frac{1}{2}\textbf{ mark}\right)$

(b) $V_1 I_1 = 10 \text{ kW}$ $\left(\frac{1}{2}\textbf{ mark}\right)$

$200 \times I_1 = 1000 \text{ W}$

$I_1 = \frac{10000}{200} = 50 \quad \text{A}$ $\left(\frac{1}{2}\textbf{ mark}\right)$

Heat loss in the primary coil is

$H = I_1^2 R = (50)^2 \times 0.2 = 500 \text{ W}$ $\left(\frac{1}{2}\textbf{ mark}\right)$

Or

(i) Instantaneous value of the emf is the value of emf at any instant of time. **(1 mark)**

(ii) Expression of emf induced across a metal conductor moving perpendicular to uniform

Field is given by e = Blv **(1 mark)**

(iii) Given,

For a series RC circuit

$I_{rms} = 1.5 \quad \text{mA} = 1.5 \times 10^{-3} \quad \text{A}$

$\omega = 100 \text{ rad s}^{-1}$

$R = 10 \quad k\Omega = 10 \times 10^3 \quad \Omega$

$C = 0.50 \quad \mu F = 0.5 \times 10^{-6} \quad F$

To find the rms voltage across the capacitor (V_C) and impedance of the circuit (Z)

We have $V_C = I_{rms} \quad X_C \ \ldots(i)$ **$\left(\frac{1}{2}\text{ mark}\right)$**

where

$X_C = \frac{1}{\omega C} = \frac{1}{(100)(0.5 \times 10^{-6})} \quad \Omega = 20000\ \Omega \ \ldots(ii)$ **$\left(\frac{1}{2}\text{ mark}\right)$**

Substituting equation (ii) in (i)

$V_C = I_{rms}\left(\frac{1}{\omega C}\right) = (1.5 \times 10^{-3})(20000) = 30\ V$ **(1 mark)**

Also impedance of the circuit

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{\left(10 \times 10^3\right)^2 + \left(20000\right)^2}\,\Omega$$

$= 2.24 \times 10^4\ \Omega$

$= 22.4\ k\Omega$ **(1 mark)**

26. (i) The phenomenon of non-uniform distribution of light energy in a medium when two or more light waves from coherent sources superpose over it is known as interference of light. **(1 mark)**

Intensity of light is the maximum where constructive interference takes place and minimum for destructive interference. Alternate bright and dark bands are obtained, due to interference of monochromatic light.

$\left(\frac{1}{2}\text{ mark}\right)$

There is only a redistribution of light intensity in the interference pattern. There is an energy transfer from regions of destructive interference to those of constructive interference. Since no energy is created or annihilated, the law of conservation of energy is followed. $\left(\frac{1}{2}\textbf{ mark}\right)$

(ii) To find out the diameter of the third bright fringe
Given, D = 1.6 m, d = 2 mm = 2×10^{-3} m,

$\lambda = 500 \text{ nm} = 500 \times 10^{-9}$ m $\left(\frac{1}{2}\textbf{ mark}\right)$

Distance to the nth bright fringe from the centre is given as

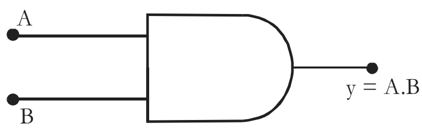
$$x_n = \frac{Dn\lambda}{d}$$ $\left(\frac{1}{2}\textbf{ mark}\right)$

Therefore $x_3 = \frac{3D\lambda}{d} = \frac{3 \times 1.6 \times 500 \times 10^{-9}}{2 \times 10^{-3}} = 1.2 \times 10^{-3}$ m $\left(\frac{1}{2}\textbf{ mark}\right)$

This gives the radius of the 3rd fringe. The diameter will be

$2x_3 = 2 \times 1.2 \times 10^{-3}$ m = 2.4×10^{-3}m $\left(\frac{1}{2}\textbf{ mark}\right)$

27. AND gate is a gate with two or more inputs and one output. Also it gives its output only when all its inputs are present.
The symbol is given below



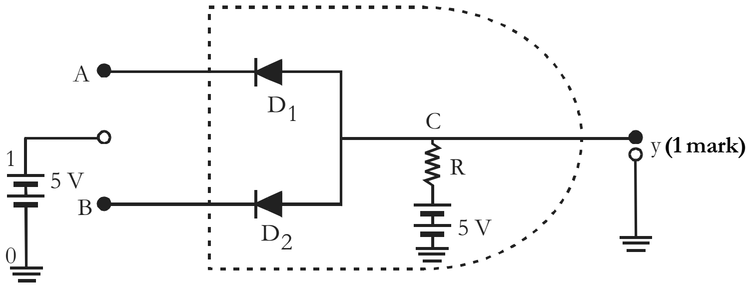
$\left(\frac{1}{2}\textbf{ mark}\right)$

Its logical operations can be summarized with the help of the truth table.

A	B	y = A . B
0	0	0
0	1	0
1	0	0
1	1	1

(1 mark)

The AND gate can be realized by an electronic circuit making use of two ideal junction diodes D_1 and D_2 as shown in the figure below.



(i) A = 0 and B = 0. Both the diodes D_1 and D_2 get forward biased and hence conduct. No voltage drop occurs across D_1 or D_2. Hence, the output y is 0.
(ii) A = 0 and B = 1. D_1 will conduct, D_2 not. Now, output y is voltage drop across D_2, which is 0.
(iii) A = 1 and B = 0. D_2 will conduct and D_1 not. Now, output y is voltage across D_1 and hence it is 0.
(iv) A = 1 and B = 1. D_1 and D_2 will not conduct. The output y will be 1.

$$\left(\frac{1}{2} \times 4 = 2 \text{ marks}\right)$$

Chemistry

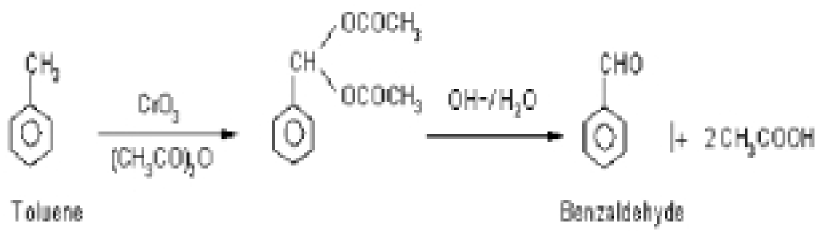
1. When a constituent particle is present on the edge of a unit cell, then it is shared by four unit cells. **(1 mark)**

2. i. First order reaction $\left(\frac{1}{2}\textbf{ mark}\right)$

ii. $-\frac{k}{2.303}$ $\left(\frac{1}{2}\textbf{ mark}\right)$

3. The IUPAC name of $(CH_2CN)_2$ is ethanedinitrile. **(1 mark)**

4.



(1 mark)

5. This is because salt water is hypertonic solution, which causes fluids causing irritation to come out.

(1 mark)

6. (i) Dispersion forces $\left(\frac{1}{2}\textbf{ mark}\right)$

(ii) ion – induced dipolar attraction $\left(\frac{1}{2}\textbf{ mark}\right)$

(iii) ion – dipole attraction $\left(\frac{1}{2}\ \textbf{mark}\right)$

(iv) H-bonding $\left(\frac{1}{2}\ \textbf{mark}\right)$

7. (i) Sublimation of ammonium chloride:

In this process, solid ammonium chloride is converted to gaseous state, thus the change in entropy is positive (because gas has more randomness than solid). **(1 mark)**

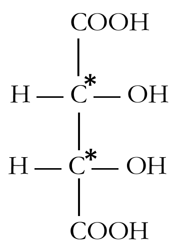
(ii) $NH_4NO_3(s) \xrightarrow{\Delta} N_2O(g) + 2H_2O(g)$

In this reaction, the reactant which is in solid state is converted to gaseous products. Thus, the change in entropy is positive. **(1 mark)**

8. The electron deficiency of boron atom is reduced due to pπ–pπ bond which it forms due to its small size. **(1 mark)**

Aluminium because of its large size does not form pπ –pπ bond. $AlCl_3$ makes up its electron deficiency by bridging with another molecule of $AlCl_3$ i.e., it exists as a dimmer. **(1 mark)**

9.

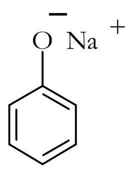


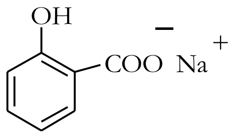
(1 mark)

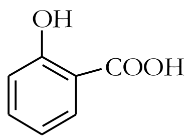
meso-2,3-dihydroxybutanoic acid

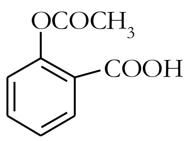
Asymmetric carbon is marked with asterisk. **(1 mark)**

10.

A is  $\left(\frac{1}{2}\ \textbf{mark}\right)$

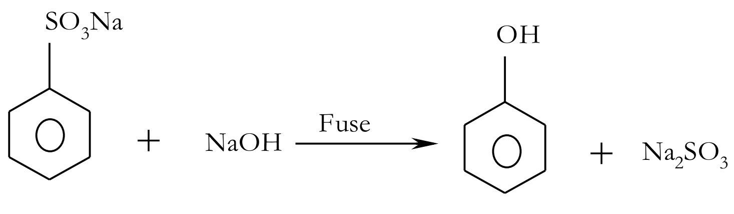
B is  $\left(\frac{1}{2}\ \textbf{mark}\right)$

C is  $\left(\frac{1}{2}\ \textbf{mark}\right)$

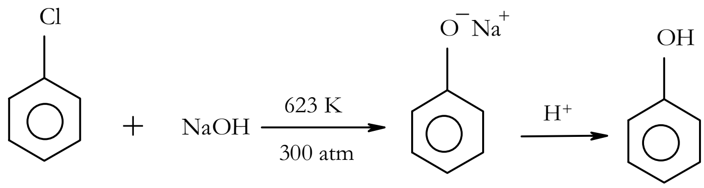
D is  $\left(\frac{1}{2}\ \textbf{mark}\right)$

Or

10.



(1 mark)



(1 mark)

11. Monomer is the simple molecule or structural unit whereas polymer is made up of a large number of monomer units joined together through covalent bonds. **(1 mark)**

The monomer unit (units) of:

i. Nylon 66 is hexamethylene diamine and adipic acid $\left(\frac{1}{4}+\frac{1}{4}\text{ mark}\right)$

ii. Nylon 6 is caprolactam $\left(\frac{1}{2}\text{ mark}\right)$

12. A pair of stereoisomers which differ in configuration only around C_1 carbon are called anomers and the C_1 carbon is called the glycosidic carbon. $\left(1\frac{1}{2}\text{ marks}\right)$

Such a carbon is also known as anomeric carbon. $\left(\frac{1}{2}\text{ mark}\right)$

13. MO configuration of O_2^+ (15 electrons) is

$$[\sigma(1s)]^2\,[\sigma^*(1s)]^2\,[\sigma(2s)]^2\,[\sigma^*(2s)]^2\,[\sigma(2p_z)]^2\,[\pi(2p_x)]^2 = [\pi(2p_y)]^2\,[\pi^*(2p_x)]^1$$

$\left(\frac{1}{2}\text{ mark}\right)$

Bond order of $O_2^+ = \frac{N_b - N_a}{2}$

$= \frac{10-5}{2} = 2.5$ $\left(\frac{1}{2}\textbf{ mark}\right)$

MO configuration of O_2^- (17 electrons) is

$$[\sigma(1s)]^2 [\sigma^*(1s)]^2 [\sigma(2s)]^2 [\sigma^*(2s)]^2 [\sigma(2p_z)]^2 [\pi(2p_x)]^2 = [\pi(2p_y)]^2 [\pi^*(2p_x)]^2 = [\pi^*(2p_y)]^1$$

$\left(\frac{1}{2}\textbf{ mark}\right)$

Bond order of $O_2^- = \frac{N_b - N_a}{2}$

$= \frac{10-7}{2} = 1.5$ $\left(\frac{1}{2}\textbf{ mark}\right)$

As bond dissociation energy is directly proportional to the bond order, therefore order of bond dissociation energy is $O_2^+ > O_2^-$ **(1 mark)**

Or

Heisenberg's uncertainity principle states that it is impossible to determine both the exact velocity and exact position of small particle simultaneously with absolute accuracy. **(1 mark)**

Given,

$\Delta v = \frac{99}{100} \times 50 = 49.5 \text{ ms}^{-1}$ $\left(\frac{1}{2}\textbf{ mark}\right)$

$h = 6.6 \times 10^{-34}$ Js

$m = 9.1 \times 10^{-31}$ kg

$\Delta x \cdot \Delta p \approx \frac{h}{4\pi}$

$$\Delta x \cdot m\Delta v \approx \frac{h}{4\pi}$$ **$\left(\frac{1}{2}\text{ mark}\right)$**

$$\therefore \Delta x \times 9.1 \times 10^{-31} \times 49.5 = \frac{6.6 \times 10^{-34}}{4 \times \frac{22}{7}}$$

$$\Delta x = \frac{6.6 \times 10^{-34} \times 7}{4 \times 22 \times 9.1 \times 10^{-31} \times 49.5} = 1.17 \times 10^{-6}\ \text{m}$$ **(1 mark)**

14. i. The substance which is expected to possess paramagnetism or ferromagnetism on the basis of a large number of unpaired electrons but actually possess zero net magnetic moment (due to the presence of equal number of magnetic moments in the opposite directions) is called anti-ferromagnetic substance. **(1 mark)**

ii. In some piezoelectric crystals, the dipoles are permanently polarized even in the absence of the electric field and direction of polarization changes on application of electric field. This phenomenon is called ferroelectricity. **(1 mark)**

iii. A unit cell in which the constituent particles are present only at the corners is called a primitive unit cell (or simple cubic unit cell). **(1 mark)**

15. $\Delta T_b = K_b m$

$T_b - T_o = K_b m$

$T_b - 100 = K_b m$ (as $T_o = 100^oC$)

or $T_b = K_b m + 100$ **$\left(\frac{1}{2}\text{ mark}\right)$**

$\Delta T_f = K_f m$

$T_o - T_f = K_f m$

$0 - T_f = K_f m$ (as $T_o = 0^oC$) **$\left(\frac{1}{2}\text{ mark}\right)$**

$T_b - T_f = 105$

$100 + K_b m - (-K_f m) = 105$
$100 + 0.53\ m + 1.86\ m = 105$
$2.39\ m = 5$

$m = 2.1$ $\left(\frac{1}{2}\text{ mark}\right)$

Thus, molality = 2.1 m

$\text{Molality} = \frac{W_B}{M_B} \times \frac{1}{W_A} \times 1000$ $\left(\frac{1}{2}\text{ mark}\right)$

$2.1 = \frac{W_B}{60} \times \frac{1}{100} \times 1000$

$W_B = \frac{2.1 \times 60}{10} = 12.6\ g$ $\left(\frac{1}{2} + \frac{1}{2}\text{ mark}\right)$

16. Lets consider a general form for the rate law as:

$\text{Rate} = k[A]^a [B_2]^b \ldots (i)$ $\left(\frac{1}{2}\text{ mark}\right)$

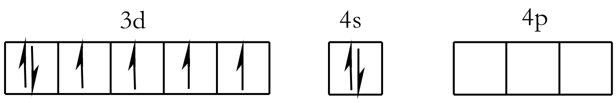
Where a and b are the order of the reaction with respect to A and B2 , respectively.
Put the value in the equation (i):
$1.30 \times 10^{-3} = k(1.0)^a (1.0)^b \ldots (ii)$
$5.20 \times 10^{-3} = k(2.0)^a (1.0)^b \ldots (iii)$
$4.16 \times 10^{-2} = k(4.0)^a (2.0)^b \ldots (iv)$
On dividing equation (iii) by equation (ii),
$4 = (2)^a$
$(2)^2 = (2)^a$
Therefore, a = 2 **(1 mark)**
Divide equation (iv) by equation (iii) and put a = 2 in equation (iv),
$8 = (2)^2 (2)^b$
$8 = 4 (2)^b$
$2 = (2)^b$
$(2)^1 = (2)^b$
Therefore, b = 1 **(1 mark)**
Put the value of a and b in equation (i)

Rate = $k[A]^2[B_2]$ $\left(\frac{1}{2}\text{ mark}\right)$

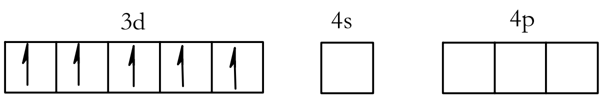
17. i. chlorobis(ethylenediamine)nitritocobalt(III) ion **(1 mark)**

ii. In $[Fe(CN)_6]^{3-}$, Fe is in +3 oxidation state.

Fe in ground state



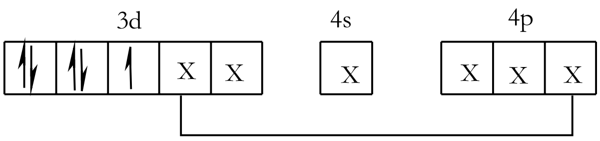
Fe^{3+}



$\left(\frac{1}{2}\text{ mark}\right)$

Since CN^- is strong field ligand, thus,

$[Fe(CN)_6]^{3-}$



$\left(\frac{1}{2}\text{ mark}\right)$

d^2sp^3

As hybridization of $[Fe(CN)_6]^{3-}$ is d^2sp^3, thus its shape is octahedral.

$\left(\frac{1}{2}\text{ mark}\right)$

The compound is paramagnetic in nature due to the presence of unpaired electron.

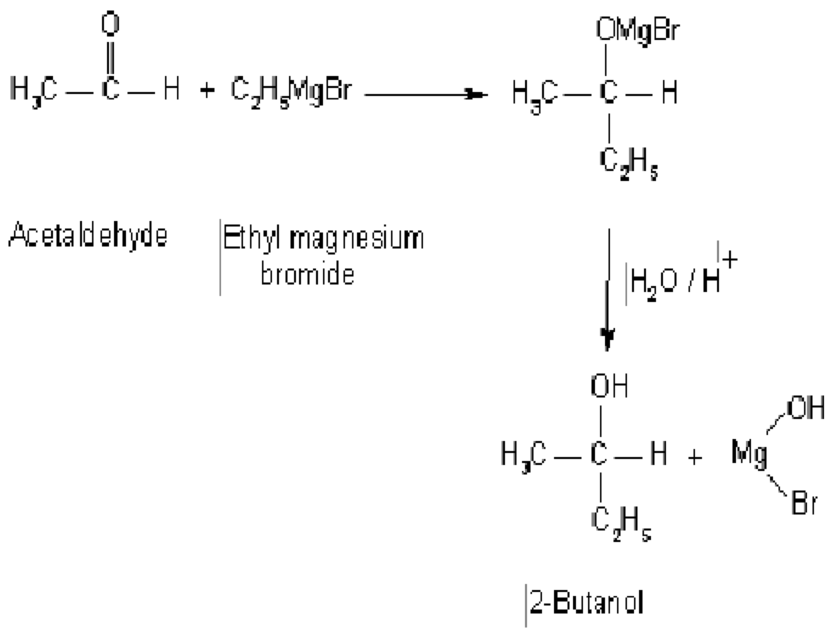
$\left(\frac{1}{2}\ \textbf{mark}\right)$

18. (i) $^{14}_{7}n + ^{1}_{0}n \rightarrow ^{14}_{6}C + ^{1}_{1}H$ **(1 mark)**

(ii) $^{234}_{90}Th \rightarrow ^{234}_{91}Pa + \beta$ **(1 mark)**

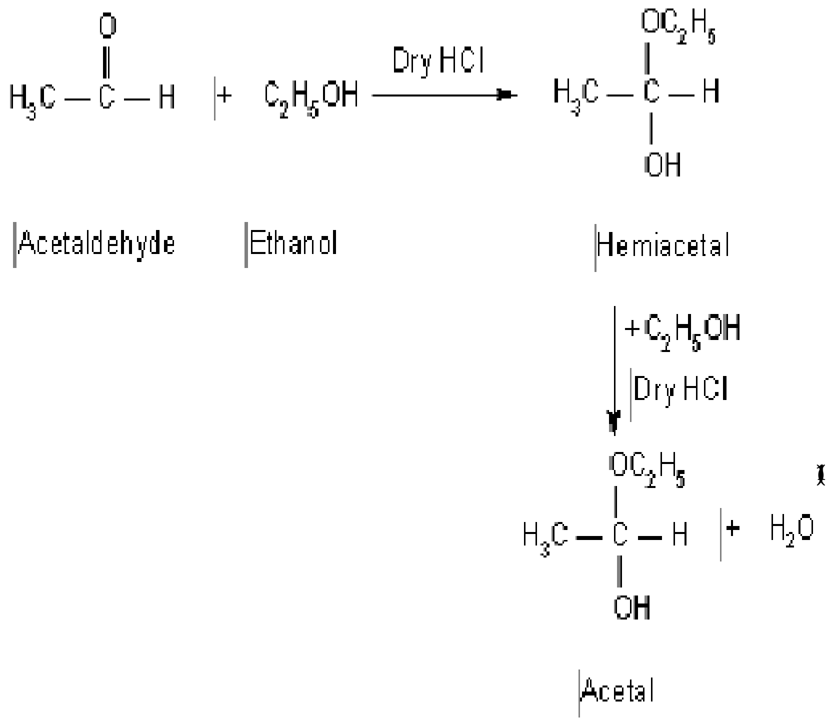
(iii) $^{133}_{56}Ba + e^{-} \rightarrow ^{133}_{57}Cs + X - ray$ **(1 mark)**

19. i. Reaction of acetaldehyde with ethyl magnesium bromide followed by hydrolysis.

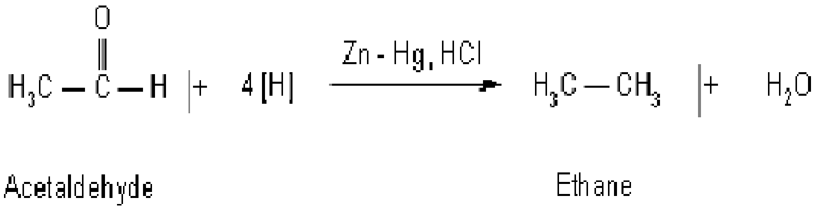


(1 mark)

ii. Reaction of acetaldehyde with ethanol in the presence of dry HCl.



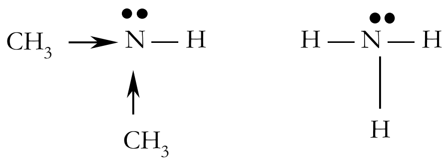
iii. Reaction of acetaldehyde with amalgamated zinc and concentrated hydrochloric



(1 mark)

20. i. Ethylamine is stronger base than ammonia. This is due to the reason that ethyl groups are electron donating in nature as a result of which electron density on nitrogen atom increases and thus they can donate the ion pair of electrons more easily than ammonia.

In ethylamine In ammonia



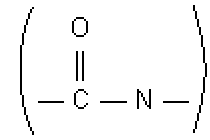
$\left(1\frac{1}{2}\ \textbf{marks}\right)$

ii. Isocyanides or carbylamines are produced by primary amines on reaction with chloroform and alcoholic KOH. These have unpleasant odours and can be used as distinguishing tests for secondary and tertiary amines, which do not give this test.

$$\underset{\text{Ethylamine}}{CH_3CH_2NH_2} + CHCl_3 + 3KOH(alc) \xrightarrow{\Delta} \underset{\text{Ethyl carbylamine}}{CH_3CH_2N \overset{\rightarrow}{=} C} + 3KCl + 3H_2O$$

$\left(1\frac{1}{2}\ \textbf{marks}\right)$

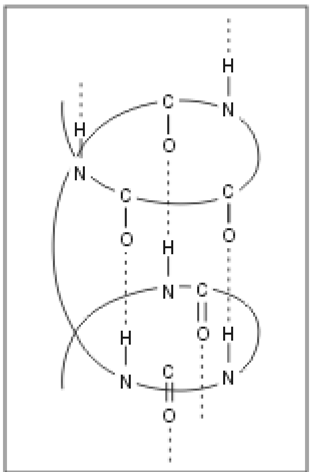
21. Protein is a polypeptide, which is made of large number of amide linkages.

The amide linkage  has oxygen and hydrogen present in it, hence have tendency to form hydrogen bonding. Therefore in a large chain polypeptide hydrogen bonding exists between different amide linkages, which give specific conformation to protein. The secondary structure of protein refers to conformation of polypeptide chain, which arises due to hydrogen bonding. **(1 mark)**

Two different secondary structures of proteins are possible depending upon the size of side chain (R).

They are:

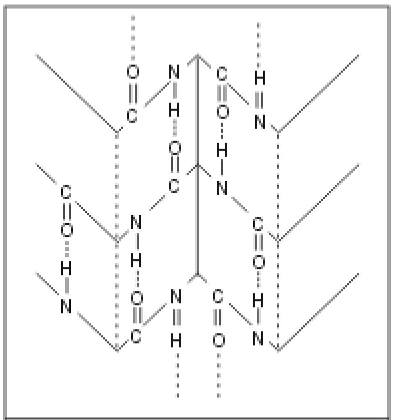
α-helix structure: If the size of the R group is quite large, the chain coils to form right handed α-helix structure.



Secondary structure of α-helix

(1 mark)

β -flat sheet: If the size of the R group is small, the polypeptide chains lie side by side in a zigzag manner to give β -pleated sheet structure.



β-pleated structure

(1 mark)

22. Cosmetic means beautification or improvement of the complexion of skin. And creams are used for the facial make-up. **(1 mark)**

Creams are classified as:

i. Cleansing cream: It is used for removing make-up, oil, etc. from face. $\left(\frac{1}{2}\textbf{ mark}\right)$

ii. Cold cream: It is used to prevent roughness and chaffing of skin. $\left(\frac{1}{2}\textbf{ mark}\right)$

iii. Vanishing cream: It is used to keep skin cool and oily. $\left(\frac{1}{2}\textbf{ mark}\right)$

iv. Bleach cream: It is used to bleach dark skin. $\left(\frac{1}{2}\textbf{ mark}\right)$

23. For the given reaction,

$$H_2(g) + I_2(g) \longrightarrow 2HI\ (g)$$

$\Delta H^0 = 51.9$ kJ mol^{-1}= 51900 J mol^{-1}
T = 298 K

$$\Delta S^0 = \Sigma S^0(\text{products}) - \Sigma S^0(\text{reactants})$$

$$\Delta S^0 = \left[2 \times S^0(HI)\right] - \left[S^0(H_2) + S^0(I)_2\right]$$

$$\Delta S^0 = [2 \times 206.3] - [130.6 + 116.7]$$

$\Delta S^0 = 412.6 - 247.3$ **(1 mark)**

$$\Delta S^0 = 165.3\ JK^{-1}\ mol^{-1}$$

$$\Delta G^0 = \Delta H^0 - T\Delta S^0$$

$$\Delta G^0 = 51900 - 298 \times 165.3$$

$$\Delta G^0 = 51900 - 49259.4$$

$\Delta G^0 = 2640.6$ J mol–1 **(1 mark)**

Since ΔG^0 is positive, the reaction is not feasible in the standard state at 298 K temperature. **(1 mark)**

24. **i.** According to Hardy Schulze law, greater the valency of oppositely charged ion of the electrolyte being added, faster is the coagulation. **(1 mark)**
Thus order of ions for the coagulation of negatively charged As_2S_3 sol is $Al^{3+} > Ba^{2+} > Na^+$

(1 mark)

ii. Dialysis helps in removing undesirable ions from a colloidal preparation thereby helping to stabilize the colloid. **(1 mark)**

25. Symbolic representation of the electrochemical cell is:

$Cu / Cu^{2+} || Ag^{+} / Ag$ $\left(\frac{1}{2}\textbf{ mark}\right)$

Oxidation takes place at Cu electrode and reduction at Ag electrode.

$\left(\frac{1}{2}+\frac{1}{2}\textbf{ mark}\right)$

Electrons move from Cu electrode to Ag electrode. $\left(\frac{1}{2}\textbf{ mark}\right)$

At anode, the following reaction takes place,

$Cu \longrightarrow Cu^{2+} + 2e^{-}$ (Oxidation) $\left(\frac{1}{2}\textbf{ mark}\right)$

At cathode, the following reaction takes place,

$2Ag^{+} + 2e^{-} \longrightarrow 2Ag$ (Reduction) $\left(\frac{1}{2}\textbf{ mark}\right)$

ii. $Cu\,(s) + 2Ag^{+}(aq) \longrightarrow Cu^{2+}(aq) + 2Ag(s)$

$$E_{cell} = E^{0}_{cell} - \frac{0.0591}{n} \log \frac{\left[Cu^{2+}\right]}{\left[Ag^{+}\right]}$$ $\left(\frac{1}{2}\textbf{ mark}\right)$

$$E^{0}_{cell} = E^{0}_{Ag^{+}/Ag} - E^{0}_{Cu^{2+}/Cu} = 0.80 - (0.34)$$

$$= 0.46V$$ $\left(\frac{1}{2}\textbf{ mark}\right)$

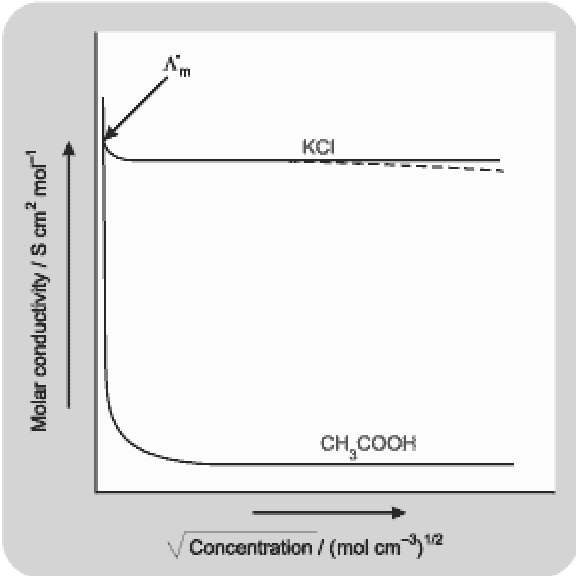
$$\therefore 0 = 0.46 - \frac{0.0591}{2} \log \frac{(0.1)}{\left[Ag^{+}\right]^{2}} \; ; \; 0.46 = \frac{0.0591}{2} \log \frac{(0.1)}{[Ag^{+}]^{2}}$$

$$\frac{(0.1)}{[Ag^{+}]^{2}} = \text{anti}\log(15.57) = 3.71 \times 10^{15} \; ; \; [Ag^{+}]^{2} = 2.7 \times 10^{-17}$$

$$[Ag^{+}] = \sqrt{2.7 \times 10^{-17}} = 5.2 \times 10^{-9}\ M$$ **(1 mark)**

Or

25. i. The molar conductivity of both strong and weak electrolytes generally increases with dilution. The variation in molar conductivity for a strong electrolyte (such as KCl) and a weak electrolyte (such as CH_3COOH) can be shown by the following plot.



(1 mark)

The curve obtained for a strong electrolyte shows that there is a small increase in conductance with dilution. This is because a strong electrolyte is completely ionized in solution and thus, there is a very small increase due to decreased attraction between the ions. The value of molar conductivity attains a maximum value at infinite dilution. This maximum value is called molar conductivity at infinitedilution.

The curve obtained for a weak electrolyte shows that in very dilute solutions, there is a very large increase in conductance with dilution. This is because a weak electrolyte ionizes to a small extent in concentrated solutions and as the concentration of the electrolyte is reduced (i.e. with dilution), the extent of ionization increases. This increases the number of ions available for conduction in solution thereby accounting for a very large increase in the molar conductivity with dilution. $\left(1\frac{1}{2}\text{ marks}\right)$

ii. l = 1 cm
A = 2 cm^2
R = 166.5 ohms

Normality = $\frac{1}{50}$

Equivalent conductivity, $\Lambda_{eq} = \kappa \times \frac{1000}{\text{Normality}}$ $\left(\frac{1}{2}\text{ mark}\right)$

But $\kappa = G \times \frac{l}{A}$ $\left(\frac{1}{2}\text{ mark}\right)$

$\kappa = \frac{1}{R} \times \frac{1}{A} \left(\because G = \frac{1}{R}\right)$

$\kappa = \frac{1}{166.5} \times \frac{1}{2}$ $\left(\frac{1}{2}\text{ mark}\right)$

$\Lambda_{eq} = \kappa \times \frac{1000}{\text{Normality}}$; $\Lambda_{eq} = \frac{1}{166.5} \times \frac{1}{2} \times \frac{1000}{\frac{1}{50}}$

= 150.2 $ohm^{-1}\ cm^2\ eq^{-1}$ $\left(\frac{1}{2} + \frac{1}{2}\text{ mark}\right)$

26. (i) Lanthanides and actinides are called inner transition elements. Their general electronic configuration is $(n-2)f^{1-14}$ $(n-1)^{d0}$ or 1^{ns2}.

(1 mark)

(ii) The typical oxidation state of lanthanides is + 3. This oxidation state is commonly attained because lanthanides have two electrons in ns-orbital and one electron in (n - 1)d-orbital which can be lost. Eu, Sm, etc., show an oxidation state of + 2 because of non-availability of (n - 1)d electron in their electronic configurations. **(1 mark)**

(iii) In transition elements, the (n - 1)d orbital is filled up successively while for inner transition elements, (n - 2)f-orbitals are occupied by the last incoming electron. Thus, transition elements have two incomplete shells namely (n - 1)d and ns while inner transition elements have three incomplete shells, (n - 2)f, (n - 1)d and ns. **(1 mark)**

vi. (a) $K_2Cr_2O_7 + 7H_2SO_4 + 6KI$

$\longrightarrow 4K_2SO_4 + Cr_2(SO4)_3 + 3I_2 + 7H_2O$ **(1 mark)**

(b) $2\ MnO^{4-} + 10\ Fe^{2+} + 16\ H^+ \longrightarrow 2\ Mn^{2+} + 10\ Fe^{3+} + 8H_2O$

(1 mark)

27. i. Out of HF, HCl, HBr and HI, HF is the weakest acid. $\left(\frac{1}{2}\textbf{ mark}\right)$

This is due to the high bond dissociation energy of HF with the result that tendency to release protons of HF is less than that of HCl, HBr or HI.

(1 mark)

ii. Bleaching powder is prepared by passing dry chlorine over heated slaked lime.

$$Ca(OH)_2 + Cl_2 \xrightarrow{\Delta} CaOCl_2 + H_2O$$

(1 mark)

Bleaching powder is used:

a. As a bleaching agent for wool and linen goods.
b. For making wool unshrinkable.
c. As a disinfectant and germicide.

(any two) $\left(\frac{1}{4}+\frac{1}{4}\textbf{ mark}\right)$

iii. Examples of compounds in which halogens show different positive oxidation states.

Example	Oxidation state of chlorine
Cl_2O	+1
ClO_2	+4
Cl_2O_3	+3
Cl_2O_7	+7

$\left(\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\textbf{ mark}\right)$

Mathematics

Section – A

1. We have $A = \begin{bmatrix} 1 & 0 \\ -1 & 6 \end{bmatrix}$

$\therefore A^2 = AA = \begin{bmatrix} 1 & 0 \\ -1 & 6 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 6 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 1+0 & 0+0 \\ -1-6 & 0+36 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 1 & 0 \\ -7 & 36 \end{bmatrix}$ **(1 mark)**

And $7A + \lambda I = 7\begin{bmatrix} 1 & 0 \\ -1 & 6 \end{bmatrix} + \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 7 & 0 \\ -7 & 42 \end{bmatrix} + \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

$= \begin{bmatrix} 7+\lambda & 0 \\ -7 & 42+\lambda \end{bmatrix}$ **(1 mark)**

$\therefore A^2 = 7A + \lambda I$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ -7 & 36 \end{bmatrix} = \begin{bmatrix} 7+\lambda & 0 \\ -7 & 42+\lambda \end{bmatrix}$ $\left(\frac{1}{2}\textbf{ mark}\right)$

$\Rightarrow 1 = 7 + \lambda$ and $36 = 42 + \lambda$

$\Rightarrow \lambda = -6$ $\left(\frac{1}{2}\textbf{ mark}\right)$

Or

Here, $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 5 \\ 1 & 2 \end{bmatrix}$,

$$(AB) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 9 \\ 16 & 23 \end{bmatrix}$$ **(1 mark)**

$$\text{LHS} = (AB)' = \begin{bmatrix} 6 & 16 \\ 9 & 23 \end{bmatrix}$$ $\left(\frac{1}{2}\text{ mark}\right)$

$$\text{RHS} = B'A' = \begin{bmatrix} 4 & 1 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$ $\left(\frac{1}{2}\text{ mark}\right)$

$$= \begin{bmatrix} 4+2 & 12+4 \\ 5+4 & 15+8 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 16 \\ 9 & 23 \end{bmatrix} = \text{LHS}$$ **(1 mark)**

Hence proved.

2. $$\text{LHS} = \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix}$$

$$= xyz \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$$ [Taking out x from C_1, y from C_2 and z from C_3]

$\left(\frac{1}{2}\text{ mark}\right)$

$$= xyz \begin{vmatrix} 1 & 0 & 0 \\ x & y-z & z-x \\ x^2 & y^2-z^2 & z^2-x^2 \end{vmatrix}$$

[Applying $C_2 \to C_2 - C_3$ and $C_3 \to C_3 - C_1$] **(1 mark)**

$$= xyz(y-z)(z-x)\begin{vmatrix} 1 & 0 & 0 \\ x & 1 & 1 \\ x^2 & y+z & z+x \end{vmatrix}$$

[Taking out (y – z) from C_2 and (z – x) from C_3] $\left(\frac{1}{2}\textbf{ mark}\right)$

$$= xyz(y-z)(z-x)\begin{vmatrix} 1 & 0 & 0 \\ x & 1 & 0 \\ x^2 & y+z & x-y \end{vmatrix}$$ [Applying $C_3 \to C_3 - C_2$] $\left(\frac{1}{2}\textbf{ mark}\right)$

$= xyz(y-z)(z-x)[x-y-0]$ [Expanding along the first row]

$= xyz(x-y)(y-z)(z-x)$ $\left(\frac{1}{2}\textbf{ mark}\right)$

3. By definition of conditional probability, we have

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$\Rightarrow P(A \cap B) = P(A) \cdot P(B/A)$

$\Rightarrow P(A \cap B) = 0.4 \times 0.6 = 0.24$ **(1 mark)**

$$\therefore P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$\Rightarrow P(A/B) = \frac{0.24}{0.8} = 0.3$ **(1 mark)**

And, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = 0.4 + 0.8 - 0.24$

$\Rightarrow P(A \cup B) = 1.2 - 0.24 = 0.96$ **(1 mark)**

4. Total number of balls = 7 + 4 = 11

Let S be the sample space, then

n(S) = number of ways of selecting 3 out of 11 balls.

$= {}^{11}C_3 = \frac{11\times10\times9}{3\times2\times1} = 165$ **(1 mark)**

Let A be the event of selecting one red and 2 white balls out of 7 red and 4 white balls.

$\therefore n(A) = {}^{7}C_1 \times {}^{4}C_2 = 7\times6 = 42$ **(1 mark)**

$\therefore$ P(getting 1 red and 2 white balls)

$= P(A)$

$= \frac{n(A)}{n(S)} = \frac{42}{165}$ **(1 mark)**

5. Let $I = \int \frac{1}{\sin(x-a)\cos(x-b)}dx$

$= \frac{1}{\cos(a-b)}\int \frac{\cos(a-b)}{\sin(x-a)\cos(x-b)}dx$ $\left(\frac{1}{2}\text{ mark}\right)$

$= \frac{1}{\cos(a-b)}\int \frac{\cos[(x-b)-(x-a)]}{\sin(x-a)\cos(x-b)}dx$ $\left(\frac{1}{2}\text{ mark}\right)$

$= \frac{1}{\cos(a-b)}\int \frac{\cos(x-a)\cos(x-b)+\sin(x-a)\sin(x-b)}{\sin(x-a)\cos(x-b)}dx$ $\left(\frac{1}{2}\text{ mark}\right)$

$= \frac{1}{\cos(a-b)}\int[\cot(x-a)+\tan(x-b)]dx$ $\left(\frac{1}{2}\text{ mark}\right)$

$= \frac{1}{\cos(a-b)}\left[\log|\sin(x-a)| - \log|\cos(x-b)|\right] + C$ $\left(\frac{1}{2}\text{ mark}\right)$

$= \frac{1}{\cos(a-b)}\log\left|\frac{\sin(x-a)}{\cos(x-b)}\right| + C$ $\left(\frac{1}{2}\text{ mark}\right)$

6. Let $I=\int\frac{\cos x}{(1+\sin x)(2+\sin x)}dx$

Let sinx = t

$\therefore \cos x dx = dt$

$\therefore I=\int\frac{1}{(1+t)(2+t)}dt$ **$\left(\frac{1}{2}\text{ mark}\right)$**

Let $\frac{1}{(1+t)(2+t)}=\frac{A}{(1+t)}+\frac{B}{(2+t)}$

$\Rightarrow 1=A(2+t)+B(1+t)$ … (i)

Substituting t = –1 in (i), we get

A = 1

Substituting t = –2 in (i), we get

B = –1

$\therefore \frac{1}{(1+t)(2+t)}=\frac{1}{(1+t)}-\frac{1}{(2+t)}$ **(1 mark)**

$\therefore I=\int\left[\frac{1}{(1+t)}-\frac{1}{(2+t)}\right]dt$

$\Rightarrow I=\log|1+t|-\log|2+t|+C$ **(1 mark)**

$\Rightarrow I=\log\left|\frac{1+t}{2+t}\right|+C$

$\Rightarrow I=\log\left|\frac{1+\sin x}{2+\sin x}\right|+C$ $[\because t=\sin x]$ **$\left(\frac{1}{2}\text{ mark}\right)$**

7. Given differential equation is

$\cos x(1+\cos y)dx-\sin y(1+\sin x)dy=0$

$\Rightarrow \frac{\cos x}{(1+\sin x)}dx=\frac{\sin y}{(1+\cos y)}dy$ [Variable separable form] **(1 mark)**

Integrating both sides, we get

$\int\frac{\cos x}{(1+\sin x)}dx=-\int\frac{-\sin y}{(1+\cos y)}dy$

$\Rightarrow \log|1+\sin x| = -\log|1+\cos y| + \log C$ $\quad \left[\because \int \frac{f'(x)}{f(x)}dx = \log[f(x)]\right]$

(1 mark)

$\Rightarrow \log|1+\sin x| + \log|1+\cos y| = \log C$

$\Rightarrow \log\left(|1+\sin x| \cdot |1+\cos y|\right) = \log C$

$\Rightarrow |1+\sin x| \cdot |1+\cos y| = C$

$\Rightarrow (1+\sin x)(1+\cos y) = C$ **(1 mark)**

8. Given differential equation is

$\left(x+2y^2\right)\frac{dy}{dx} = y$

$\Rightarrow \frac{dx}{dy} = \frac{x+2y^2}{y}$

$\Rightarrow \frac{dx}{dy} - \frac{1}{y}x = 2y$ **(1 mark)**

It is a linear differential equation of the form $\frac{dx}{dy} + Px = Q$, where $P = -\frac{1}{y}$ and $Q = 2y$

Integrating factor (I.F.) $= e^{\int P dy}$

$= e^{\int -\frac{1}{y}dy}$

$= e^{-\log|y|}$

$= e^{\log\left|\frac{1}{y}\right|}$

$= \frac{1}{y}$ **(1 mark)**

$\therefore$ The required solution is

$x \times \text{I.F.} = \int (Q \times \text{I.F.})\,dy + C$ $\left(\frac{1}{2}\ \textbf{mark}\right)$

$\Rightarrow x \times \frac{1}{y} = \int \left(2y \times \frac{1}{y}\right) dy + C$

$\Rightarrow \frac{x}{y} = 2\int dy + C$

$\Rightarrow \frac{x}{y} = 2y + C$

$\Rightarrow x = 2y^2 + Cy$ $\left(\frac{1}{2}\ \textbf{mark}\right)$

9. Given expression is a + a' (a + b) + bc

= a + (a'a +a'b) + bc[Distributivity of . over +] $\left(\frac{1}{2}\ \textbf{mark}\right)$

= a + (0 + a'b) + bc[Since a'a = 0] $\left(\frac{1}{2}\ \textbf{mark}\right)$

= a + a'b + bc $\left(\frac{1}{2}\ \textbf{mark}\right)$

= (a + a'b) + bc
= (a + b) + bc [Since a + a'b = a + b] **(1 mark)**

= a + (b + bc) [Associativity of +] $\left(\frac{1}{2}\ \textbf{mark}\right)$

= a + b [Since b + bc = b(1 + c) = b.1 = b] **(1 mark)**

10. LHS = $\lim_{x \to 2^-} f(x) = \lim_{x \to 2} (2x - 1) = 2 \times 2 - 1 = 3$ **(1 mark)**

RHS = $\lim_{x \to 2^+} f(x) = \lim_{x \to 2} (x + 1) = 2 + 1 = 3$ **(1 mark)**

And f(2) = k $\left(\frac{1}{2}\ \textbf{mark}\right)$

$\because$ The function f(x) is continuous at x = 2 [Given]

$\therefore \lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^+} f(x) = f(2)$ **$\left(\frac{1}{2}\text{ mark}\right)$**

$\Rightarrow 3 = 3 = k$

$\Rightarrow k = 3$ **(1 mark)**

11. We have, $y = (\sin x)^{\tan x} + (\cos x)^{\sec x}$

Let $u = (\sin x)^{\tan x}$

Taking log on both sides, we get

$\log u = \tan x \log(\sin x)$ **$\left(\frac{1}{2}\text{ mark}\right)$**

Differentiating both sides with respect to x, we get

$\frac{1}{u}\cdot\frac{du}{dx} = \tan x \times \frac{1}{\sin x} \times \cos x + \sec^2 x \times \log(\sin x)$

$\Rightarrow \frac{1}{u}\cdot\frac{du}{dx} = 1 + \sec^2 x \log(\sin x)$

$\Rightarrow \frac{du}{dx} = u\left[1 + \sec^2 x \log(\sin x)\right]$

$\Rightarrow \frac{du}{dx} = (\sin x)^{\tan x}\left[1 + \sec^2 x \log(\sin x)\right]$ ... (i) **(1 mark)**

And let $v = (\cos x)^{\sec x}$

Taking log on both sides, we get

$\log v = \sec x \log(\cos x)$ **$\left(\frac{1}{2}\text{ mark}\right)$**

Differentiating both sides with respect to x, we get

$\frac{1}{v}\frac{dv}{dx} = \sec x \times \frac{1}{\cos x} \times -\sin x + \sec x \tan x \log(\cos x)$

$\Rightarrow \frac{1}{v}\frac{dv}{dx} = -\sec x \tan x + \sec x \tan x \log(\cos x)$

$\Rightarrow \frac{dv}{dx} = v\left[\sec x \tan x \log(\cos x) - \sec x \tan x\right]$

$\Rightarrow \frac{dv}{dx} = (\cos x)^{\sec x}\left[\sec x \tan x \log(\cos x) - \sec x \tan x\right]$ ... (ii) **(1 mark)**

$\therefore y = u + v$

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$\Rightarrow \frac{dy}{dx} = (\sin x)^{\tan x}\left[1 + \sec^2 x \log(\sin x)\right] +$

$(\cos x)^{\sec x}\left[\sec x \tan x \log(\cos x) - \sec x \tan x\right]$ [From (i) and (ii)]

(1 mark)

12. We have $y = \cos^{-1}(2x) + 2\cos^{-1}\sqrt{1-4x^2}$ ...(i)

Substituting $2x = \cos\theta$ in (i), we get **(1 mark)**

$y = \cos^{-1}(\cos\theta) + 2\cos^{-1}\sqrt{1-\cos^2\theta}$ **(1 mark)**

$\Rightarrow y = \theta + 2\cos^{-1}(\sin\theta)$

$\Rightarrow y = \theta + 2\cos^{-1}\left[\cos\left(\frac{\pi}{2} - \theta\right)\right]$

$\Rightarrow y = \theta + 2\left(\frac{\pi}{2} - \theta\right)$

$\Rightarrow y = \pi - \theta$ **(1 mark)**

$\Rightarrow y = \pi - \cos^{-1}(2x)$ ...(ii)

Differentiating (ii) with respect to x, we get

$\frac{dy}{dx} = 0 - \frac{-2}{\sqrt{1-4x^2}}$

$\Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}}$ **(1 mark)**

13. We have, $f(x) = \cos\left(2x + \frac{\pi}{4}\right)$

$\therefore f'(x) = -2\sin\left(2x + \frac{\pi}{4}\right)$ **(1 mark)**

$\therefore f'(x) > 0$ when $\sin\left(2x + \frac{\pi}{4}\right) < 0$ **(1 mark)**

It happens when $\pi < \left(2x + \frac{\pi}{4}\right) < 2\pi$

i.e. when $\left(\pi - \frac{\pi}{4}\right) < 2x < \left(2\pi - \frac{\pi}{4}\right)$

i.e. when $\frac{3\pi}{8} < x < \frac{7\pi}{8}$ **(1 mark)**

$\therefore f'(x) > 0$ for all $x \in \left]\frac{3\pi}{8}, \frac{7\pi}{8}\right[$

Hence, f(x) is increasing in $\left]\frac{3\pi}{8}, \frac{7\pi}{8}\right[$ **(1 mark)**

Or

Given function $f(x) = x^3 - 2x^2 + x$

(i) f(x) being polynomial function, therefore it is continuous in [0, 1] $\left(\frac{1}{2}\text{ mark}\right)$

(ii) $f'(x) = 3x^2 - 4x + 1$, it exists for all values of $x \in [0, 1]$. Therefore, f(x) is differentiable in the interval]0, 1[$\left(\frac{1}{2}\text{ mark}\right)$

(iii) $f(0) = 0^3 - 2 \times 0^2 + 0 = 0$

$f(1) = 1^3 - 2 \times 1^2 + 1 = 0$

$\therefore f(0) = f(1) = 0$ **(1 mark)**

$\therefore$ All conditions of Rolle's theorem are satisfied.

So, there exists at least some $c \in]0, 1[$ such that f´(c) = 0 $\left(\frac{1}{2}\text{ mark}\right)$

$\Rightarrow 3c^2 - 4c + 1 = 0$

$\Rightarrow (c-1)(3c-1) = 0$

$\Rightarrow c = 1 \text{ or } c = \frac{1}{3}$

Here $\frac{1}{3} \in]0, 1[$ and c = 1 (rejected) $\notin]0, 1[$ $\left(1\frac{1}{2}\text{ marks}\right)$

Hence, Rolle's theorem is verified.

14. Let $I = \int \frac{3x+1}{(x-2)^2(x+2)} dx$

Let $\frac{3x+1}{(x-2)^2(x+2)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+2}$ $\left(\frac{1}{2}\text{ mark}\right)$

$\Rightarrow 3x + 1 = A(x-2)(x+2) + B(x+2) + C(x-2)^2$

$\Rightarrow 3x + 1 = A(x^2 - 4) + B(x+2) + C(x-2)^2$

Putting x = 2, we get
7 = 4B

$\Rightarrow B = \frac{7}{4}$

Putting x = –2, we get
–5 = 16C

$\Rightarrow C = \frac{-5}{16}$

Comparing coefficients of x^2 on both sides, we get
A + C = 0

$\Rightarrow A = -C$

$\Rightarrow A = \frac{5}{16}$ $\left(1\frac{1}{2}\text{ marks}\right)$

$$\therefore \frac{3x+1}{(x-2)^2(x+2)} = \frac{5}{16(x-2)} + \frac{7}{4(x-2)^2} - \frac{5}{16(x+2)}$$ **$\left(\frac{1}{2}\text{ mark}\right)$**

$$\therefore I = \int \frac{3x+1}{(x-2)^2(x+2)} dx$$

$$\Rightarrow I = \frac{5}{16}\int \frac{1}{(x-2)} dx + \frac{7}{4}\int \frac{1}{(x-2)^2} dx - \frac{5}{16}\int \frac{1}{(x+2)} dx$$

$$\Rightarrow I = \frac{5}{16}\log(x-2) - \frac{7}{4(x-2)} - \frac{5}{16}\log(x+2) + C$$ **$\left(1\frac{1}{2}\text{ marks}\right)$**

15. Let $I = \int_0^{\frac{\pi}{4}} \log(1+\tan x)dx$

$$\therefore I = \int_0^{\frac{\pi}{4}} \log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]dx$$

$\left[\text{Using the property } \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$ **(1 mark)**

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\left[1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right]dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\left[1+\frac{1-\tan x}{1+\tan x}\right]dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\left[\frac{1+\tan x+1-\tan x}{1+\tan x}\right]dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1+\tan x}\right)dx$$ **(1 mark)**

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \left[\log 2 - \log(1+\tan x)\right]dx$$

$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log 2dx - \int_0^{\frac{\pi}{4}} \log(1 + \tan x)dx$

$\Rightarrow I = \log 2\int_0^{\frac{\pi}{4}} dx - I$ **(1 mark)**

$\Rightarrow 2I = \log 2[x]_0^{\pi/4}$

$\Rightarrow 2I = \log 2\left[\frac{\pi}{4} - 0\right]$

$\Rightarrow I = \frac{\pi}{8}\log 2$ **(1 mark)**

16. Given system of equations is
x – 2y + z = 0
y – z = 2
2x – 3z = 10

Let $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 2 & 0 & -3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ 2 \\ 10 \end{bmatrix}$

$\therefore$ The given system in matrix form is AX = B

$\Rightarrow X = A^{-1}B$ **(1 mark)**

Now $|A| = \begin{vmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 2 & 0 & -3 \end{vmatrix}$

$= 1(-3-0) + 2(0+2) + 1(0-2)$

$= -3 + 4 - 2 = -1$

$\therefore |A| \neq 0$

$\therefore$ A is invertible. **(1 mark)**

Co-factors of the elements of $|A|$ are

$$C_{11}=\begin{vmatrix}1 & -1\\0 & -3\end{vmatrix}=-3,\quad C_{12}=-\begin{vmatrix}0 & -1\\2 & -3\end{vmatrix}=-2,\quad C_{13}=\begin{vmatrix}0 & 1\\2 & 0\end{vmatrix}=-2$$

$$C_{21}=-\begin{vmatrix}-2 & 1\\0 & -3\end{vmatrix}=-6,\quad C_{22}=\begin{vmatrix}1 & 1\\2 & -3\end{vmatrix}=-5,\quad C_{23}=-\begin{vmatrix}1 & -2\\2 & 0\end{vmatrix}=-4$$

$$C_{31}=\begin{vmatrix}-2 & 1\\1 & -1\end{vmatrix}=1,\quad C_{32}=-\begin{vmatrix}1 & 1\\0 & -1\end{vmatrix}=1,\quad C_{33}=\begin{vmatrix}1 & -2\\0 & 1\end{vmatrix}=1$$

$\left(1\frac{1}{2}\text{ marks}\right)$

$$\therefore \text{adj}A=\begin{bmatrix}-3 & -2 & -2\\-6 & -5 & -4\\1 & 1 & 1\end{bmatrix}'$$

$$=\begin{bmatrix}-3 & -6 & 1\\-2 & -5 & 1\\-2 & -4 & 1\end{bmatrix}$$ **$\left(\frac{1}{2}\text{ mark}\right)$**

Now $A^{-1}=\frac{1}{|A|}\text{adj } A=\frac{1}{-1}\begin{bmatrix}-3 & -6 & 1\\-2 & -5 & 1\\-2 & -4 & 1\end{bmatrix}$

$$=\begin{bmatrix}3 & 6 & -1\\2 & 5 & -1\\2 & 4 & -1\end{bmatrix}$$ **(1 mark)**

$\therefore X=A^{-1}B$

$$\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}3 & 6 & -1\\2 & 5 & -1\\2 & 4 & -1\end{bmatrix}\begin{bmatrix}0\\2\\10\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0+12-10 \\ 0+10-10 \\ 0+8-10 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$$

$\therefore$ x = 2, y = 0 and z = –2, is the required solution. **(1 mark)**

Or

We have,

$$D = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

Using $C_3 \to C_3 - C_2$ and $C_2 \to C_2 - C_1$, we get

$$D = \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-b \\ a^2 & b^2-a^2 & c^2-b^2 \end{vmatrix}$$

$$= 1\left[(b-a)\left(c^2-b^2\right)-(c-b)\left(b^2-a^2\right)\right]$$

$$= [(b-a)(c-b)(c+b)-(c-b)(b-a)(b+a)]$$

$$= (c-b)(b-a)[c+b-b-a]$$

$$= (c-b)(b-a)(c-a)$$

$$= (a-b)(b-c)(c-a)$$ **$\left(1\frac{1}{2}\text{ marks}\right)$**

Now,

$$D_1 = \begin{vmatrix} 1 & 1 & 1 \\ p & b & c \\ p^2 & b^2 & c^2 \end{vmatrix}$$

Using $C_3 \to C_3 - C_2$ and $C_2 \to C_2 - C_1$

$$\Rightarrow D_1 = \begin{vmatrix} 1 & 0 & 0 \\ p & b-p & c-b \\ p^2 & b^2-p^2 & c^2-b^2 \end{vmatrix}$$

$$= 1\left[(b-p)\left(c^2-b^2\right)-(c-b)\left(b^2-p^2\right)\right]$$

$$= (b-p)(c-b)(c+b)-(c-b)(b-p)(b+p)$$

$$= (b-p)(c-b)[c+b-b-p]$$

$$= (b-p)(c-b)(c-p)$$

$$= (p-b)(b-c)(c-p)$$ **(1 mark)**

$$D_2 = \begin{vmatrix} 1 & 1 & 1 \\ a & p & c \\ a^2 & p^2 & c^2 \end{vmatrix}$$

$$= (a-p)(p-c)(c-a)$$ **(1 mark)**

$$\text{and } D_3 = \begin{vmatrix} 1 & 1 & 1 \\ a & b & p \\ a^2 & b^2 & p^2 \end{vmatrix}$$

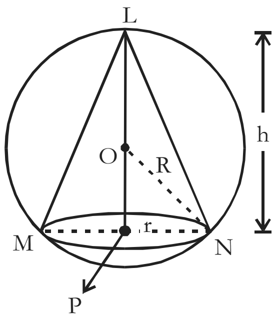
$$= (a-b)(b-p)(p-a)$$ **(1 mark)**

Now,

$$x = \frac{D_1}{D},\ y = \frac{D_2}{D} \text{ and } z = \frac{D_3}{D}$$

$$\therefore x = \frac{(p-b)(c-p)}{(a-b)(c-a)},\ y = \frac{(a-p)(p-c)}{(a-b)(b-c)} \text{ and } z = \frac{(b-p)(p-a)}{(b-c)(c-a)}$$ $\left(\mathbf{1\frac{1}{2}\ marks}\right)$

17.



Given R is the radius of the given sphere with centre O.
Let h be the height of the inscribed cone, r be the radius of the base and V be the volume.
In figure,
OP = LP – LO = h – R
In right angled triangle OPN
$ON^2 = OP^2 + PN^2$

$\Rightarrow R^2 = (h-R)^2 + r^2$ **(1 mark)**

$\Rightarrow r^2 = h(2R-h)$ … (i)

Volume of the cone, $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times h(2R-h)\times h$ [From (i)]

$\Rightarrow V = \frac{1}{3}\pi h^2(2R-h)$ … (ii) **(1 mark)**

$\therefore \frac{dV}{dh} = \frac{1}{3}\pi\left(4hR - 3h^2\right)$

$\Rightarrow \frac{dV}{dh} = \frac{1}{3}\pi h(4R-3h)$ …(iii) **(1 mark)**

For maxima or minima $\frac{dV}{dh} = 0$

$\Rightarrow \frac{1}{3}\pi h(4R-3h) = 0$

$\Rightarrow h = 0$ or $(4R - 3h) = 0$

$\Rightarrow h = 0$ or $h = \frac{4}{3}R$

$\therefore h = \frac{4}{3}R$

$[\because h \neq 0]$ **(1 mark)**

Again, $\frac{d^2V}{dh^2} = \frac{4}{3}\pi R - 2\pi h$

$\therefore \left[\frac{d^2V}{dh^2}\right]_{h=\frac{4}{3}R} = \frac{4}{3}\pi R - 2\pi \times \frac{4}{3}R = -\frac{4}{3}\pi R < 0$

$\therefore$ V is maximum when $h = \frac{4}{3}R$

i.e. 3h = 2(2R)
i.e. 3 × height = 2 × diameter **(1 mark)**

$\therefore$ Volume of the largest cone $= \frac{1}{3}\pi \times \frac{16R^2}{9} \times \left(2R - \frac{4R}{3}\right) = \frac{32}{81}\pi R^3$ **(1 mark)**

18. $y = |x+3| = \begin{cases} -(x+3) & \text{when } x < -3 \\ (x+3) & \text{when } x \geq -3 \end{cases}$ **(1 mark)**

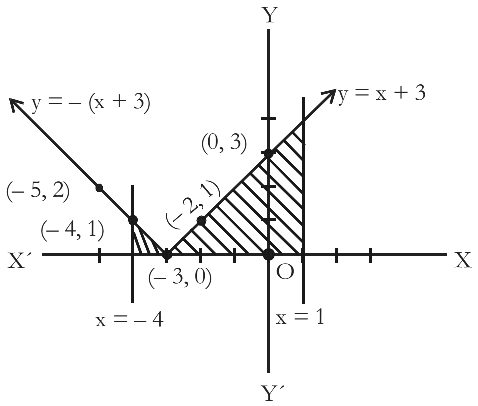
y = –x – 3 when x < –3

x	–3	–4	–5
y	0	1	2

y = x + 3 when $x \geq -3$

x	–3	–2	0
y	0	1	3

Plotting these points, we get



(1 mark)

$\therefore$ Required area = $\int_{-4}^{-3} -(x+3)dx + \int_{-3}^{1}(x+3)dx$ **$\left(1\frac{1}{2}\text{ marks}\right)$**

$= \left[-\frac{x^2}{2} - 3x\right]_{-4}^{-3} + \left[\frac{x^2}{2} + 3x\right]_{-3}^{1}$ **$\left(1\frac{1}{2}\text{ marks}\right)$**

$= \left[\left(-\frac{9}{2}+9\right) - \left(-\frac{16}{2}+12\right)\right] + \left[\left(\frac{1}{2}+3\right) - \left(\frac{9}{2}-9\right)\right]$

$= -\frac{9}{2} + 9 + \frac{16}{2} - 12 + \frac{1}{2} + 3 - \frac{9}{2} + 9$

$= \frac{17}{2}$ sq. units **(1 mark)**

Or

Let $f(x) = e^{x+2}$

a = 0, b = 3

nh = b – a = 3 – 0 = 3 **$\left(\frac{1}{2}\text{ mark}\right)$**

$\therefore \int_0^3 e^{x+2}dx$

$= \lim_{h\to 0} h\left[f(0)+f(0+h)+f(0+2h)+\ldots+f\{0+(n-1)h\}\right]$ **(1 mark)**

$= \lim_{h\to 0} h\left[e^2+e^{h+2}+e^{2h+2}+\ldots+e^{(n-1)h+2}\right]$ **(1 mark)**

$= e^2 \lim_{h\to 0} h\left[1+e^h+e^{2h}+\ldots+e^{(n-1)h}\right]$ $\left(\frac{1}{2}\ \textbf{mark}\right)$

$= e^2 \lim_{h\to 0} h\left[\frac{\left(e^h\right)^n-1}{e^h-1}\right]$ $\left(\frac{1}{2}\ \textbf{mark}\right)$

[Using formula for sum of geometric series with the first term 1 and common ratio e^h]

$= e^2 \lim_{h\to 0} h\left[\frac{e^{nh}-1}{e^h-1}\right]$

$= e^2 \lim_{h\to 0} h\left[\frac{e^3-1}{e^h-1}\right]$ $[\because nh=3]$ $\left(\frac{1}{2}\ \textbf{mark}\right)$

$= e^2 \lim_{h\to 0} \frac{h\left(e^3-1\right)}{\left\{1+h+\frac{h^2}{2!}+\frac{h^3}{3!}+\ldots\right\}-1}$ **(1 mark)**

$= e^2 \lim_{h\to 0} \frac{e^3-1}{1+\frac{h}{2!}+\frac{h^2}{3!}+\ldots}$

$= e^2\left(e^3-1\right)$ **(1 mark)**

Section – B

19. Given vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = 4\hat{i} - 4\hat{j} + 7\hat{k}$

The projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$ **(1 mark)**

$$= \frac{\left(\hat{i}-2\hat{j}+\hat{k}\right)\cdot\left(4\hat{i}-4\hat{j}+7\hat{k}\right)}{\sqrt{(4)^2+(-4)^2+(7)^2}}$$ **(1 mark)**

$$= \frac{4+8+7}{\sqrt{16+16+49}}$$

$$= \frac{19}{\sqrt{81}}$$

$$= \frac{19}{9}$$ **(1 mark)**

20. Given the position vectors of the points A, B, C and D are

$3\hat{i}-2\hat{j}-\hat{k}$, $2\hat{i}+3\hat{j}-4\hat{k}$, $\hat{i}-\hat{j}+2\hat{k}$ and $5\hat{i}-4\hat{j}+\lambda\hat{k}$ respectively.

$\therefore \overrightarrow{AB} =$ Position vector of B – Position vector of A

$$= \left(2\hat{i}+3\hat{j}-4\hat{k}\right)-\left(3\hat{i}-2\hat{j}-\hat{k}\right) = -\hat{i}+5\hat{j}-3\hat{k},$$ $\left(\frac{1}{2}\textbf{ mark}\right)$

$\overrightarrow{AC} =$ Position vector of C – Position vector of A

$$= \left(\hat{i}-\hat{j}+2\hat{k}\right)-\left(3\hat{i}-2\hat{j}-\hat{k}\right) = -2\hat{i}+\hat{j}+3\hat{k}$$ $\left(\frac{1}{2}\textbf{ mark}\right)$

And $\overrightarrow{AD} =$ Position vector of D – Position vector of A

$$= \left(5\hat{i}-4\hat{j}+\lambda\hat{k}\right)-\left(3\hat{i}-2\hat{j}-\hat{k}\right) = 2\hat{i}-2\hat{j}+(\lambda+1)\hat{k}$$ $\left(\frac{1}{2}\textbf{ mark}\right)$

Since the points A, B, C and D are coplanar.

$$\therefore \left[\overrightarrow{AB}\quad \overrightarrow{AC}\quad \overrightarrow{AD}\right] = 0$$ $\left(\frac{1}{2}\textbf{ mark}\right)$

$$\Rightarrow \begin{vmatrix} -1 & 5 & -3 \\ -2 & 1 & 3 \\ 2 & -2 & (\lambda+1) \end{vmatrix} = 0$$ **$\left(\frac{1}{2}\text{ mark}\right)$**

$\Rightarrow -1(\lambda+1+6)-5(-2\lambda-2-6)-3(4-2)=0$

$\Rightarrow -\lambda-7+10\lambda+40-6=0$

$\Rightarrow 9\lambda=-27$

$\Rightarrow \lambda=-3$ **$\left(\frac{1}{2}\text{ mark}\right)$**

21. Speed of the man in the train relative to goods train

$=\frac{300}{18}\text{ m/s}$ **(1 mark)**

$=60\text{ km/hr}$ ...(i)

Let the speed of goods train $=\text{u km/hr}$

Speed of man in passenger train relative to goods train $=50+\text{u}$ **(1 mark)**

$\Rightarrow 50+\text{u}=60$ [From (i)] **$\left(\frac{1}{2}\text{ mark}\right)$**

$\Rightarrow \text{u}=60-50=10\text{ km/hr}$ **$\left(\frac{1}{2}\text{ mark}\right)$**

So, speed of goods train $=10\text{ km/hr}$

22. Greatest height

$=\frac{\text{u}^2\sin^2\alpha}{2\text{g}}=\frac{90\times 90\times \sin^2 60°}{2\times 10}$ **$\left(\frac{1}{2}\text{ mark}\right)$**

$$=\frac{90\times 90\times\left(\frac{\sqrt{3}}{2}\right)^2}{2\times 10}$$

$=303.75\text{ m}$ **$\left(\frac{1}{2}\text{ mark}\right)$**

Time of flight $= \frac{2u\sin\alpha}{g}$ $\left(\frac{1}{2}\textbf{ mark}\right)$

$= \frac{2\times 90\times \sin 60°}{10} = \frac{2\times 90\times \frac{\sqrt{3}}{2}}{10}$

$= 9\sqrt{3}$ sec $\left(\frac{1}{2}\textbf{ mark}\right)$

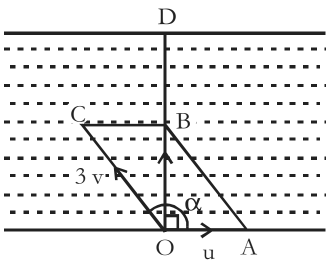
Horizontal range $= \frac{u^2\sin 2\alpha}{g}$ $\left(\frac{1}{2}\textbf{ mark}\right)$

$= \frac{90\times 90\times \sin(2\times 60)°}{10} = \frac{90\times 90\times \sin 120°}{10}$

$= \frac{90\times 90 \sin(180° - 60°)}{10} = \frac{90\times 90\times \sin 60°}{10}$

$= \frac{90\times 90\times \frac{\sqrt{3}}{2}}{10} = 405\sqrt{3}$ m $\left(\frac{1}{2}\textbf{ mark}\right)$

Or



Since the man crosses the river in a straight line (perpendicular to the stream)

∴ The direction of the resultant velocity is along OD. $\left(\frac{1}{2}\textbf{ mark}\right)$

$\therefore \tan 90^\circ = \dfrac{3u \sin\alpha}{u + 3u\cos\alpha}$ $\left(\dfrac{1}{2}\textbf{ mark}\right)$

$\Rightarrow \infty = \dfrac{3u \sin\alpha}{u + 3u\cos\alpha}$ $\left(\dfrac{1}{2}\textbf{ mark}\right)$

$\Rightarrow u + 3u\cos\alpha = 0$ $\left(\dfrac{1}{2}\textbf{ mark}\right)$

$\Rightarrow \cos\alpha = \dfrac{-1}{3}$ $\left(\dfrac{1}{2}\textbf{ mark}\right)$

$\Rightarrow \alpha = \cos^{-1}\left(\dfrac{-1}{3}\right)$ $\left(\dfrac{1}{2}\textbf{ mark}\right)$

23. The given equation of the sphere is
$2(x - 5)(x + 1) + 2(y + 5)(y - 1) + 2(z - 2)(z + 2) - 7 = 0$
$\Rightarrow 2(x^2 - 4x - 5) + 2(y^2 + 4y - 5) + 2(z^2 - 4) - 7 = 0$
$\Rightarrow 2x^2 + 2y^2 + 2z^2 - 8x + 8y - 35 = 0$

$\Rightarrow x^2 + y^2 + z^2 - 4x + 4y - \dfrac{35}{2} = 0$ **(1 mark)**

The coordinates of the centre of the sphere are

$\left[-\dfrac{1}{2}\text{ Coefficient of x}, -\dfrac{1}{2}\text{Coefficient of y}, -\dfrac{1}{2}\text{Coefficient of z}\right]$ **(1 mark)**

$\Rightarrow \left(-\dfrac{1}{2}\times -4, -\dfrac{1}{2}\times 4, -\dfrac{1}{2}\times 0\right)$

$\Rightarrow (2, -2, 0)$ $\left(\dfrac{1}{2}\textbf{ mark}\right)$

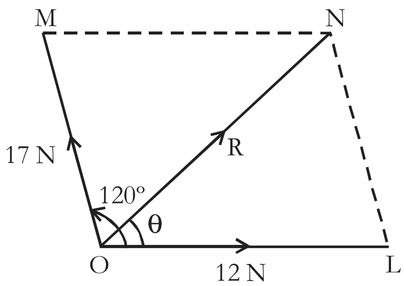
Radius of the sphere

$= \sqrt{\left(\dfrac{1}{2}\text{Coefficient of x}\right)^2 + \left(\dfrac{1}{2}\text{Coefficient of y}\right)^2 + \left(\dfrac{1}{2}\text{Coefficient of z}\right)^2 - \text{Constant term}}$ **(1 mark)**

$$= \sqrt{(-2)^2 + (2)^2 + (0)^2 - \left(-\frac{35}{2}\right)}$$

$$= \sqrt{4 + 4 + \frac{35}{2}} = \sqrt{\frac{51}{2}} \text{ Units}$$ $\left(\frac{1}{2}\textbf{ mark}\right)$

24.



Let R be the resultant of two forces 12 N and 17 N acting at an angle of 120°

$$\therefore R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$ $\left(\frac{1}{2}\textbf{ mark}\right)$

$$\Rightarrow R = \sqrt{(12)^2 + (17)^2 + 2\times12\times17\cos120°}$$ **(1 mark)**

$$\Rightarrow R = \sqrt{(12)^2 + (17)^2 + 2\times12\times17\cos(180° - 60°)}$$

$$\Rightarrow R = \sqrt{144 + 289 - 2\times12\times17\times\frac{1}{2}}$$

$$\Rightarrow R = \sqrt{144 + 289 - 204}$$

$$\Rightarrow R = \sqrt{229} \text{ N}$$ $\left(\frac{1}{2}\textbf{ mark}\right)$

Let θ be the angle which resultant R makes with 12 N.

$$\therefore \tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$ $\left(\frac{1}{2}\textbf{ mark}\right)$

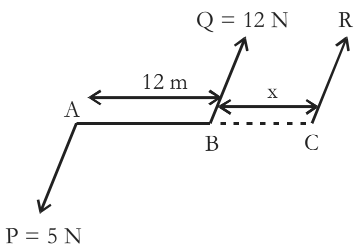
$= \dfrac{17 \times \sin 120°}{12 + 17 \times \cos 120°}$ **(1 mark)**

$= \dfrac{17 \times \dfrac{\sqrt{3}}{2}}{12 + 17 \times -\dfrac{1}{2}}$

$= \dfrac{17\sqrt{3}}{24 - 17} = \dfrac{17\sqrt{3}}{7}$

$\Rightarrow \theta = \tan^{-1}\left(\dfrac{17\sqrt{3}}{7}\right)$ $\left(\dfrac{1}{2}\text{ mark}\right)$

Or



Let resultant R acts at C on produced AB at a distance x from B.
Since Q > P

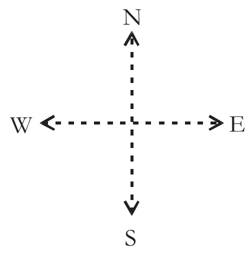
$\therefore$ Resultant $R = Q - P$ [Unlike parallel forces] **(1 mark)**

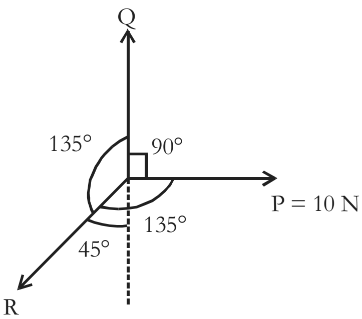
$= 12 - 5 = 7$ N $\left(\dfrac{1}{2}\text{ mark}\right)$

$P \times AC = Q \times BC$ **(1 mark)**

$\Rightarrow 5 \times (12 + x) = 12 \times x$ **(1 mark)**

$\Rightarrow x = \dfrac{60}{7}$ $\left(\dfrac{1}{2}\text{ mark}\right)$

25. 



According to the problem, angle between P and Q, Q and R and P and R are 90°, 135° and 135° respectively.
Using Lami's theorem, we get

$$\frac{10}{\sin 135°} = \frac{Q}{\sin 135°} = \frac{R}{\sin 90°}$$ [Given P = 10 N] **(1 mark)**

$$\Rightarrow \frac{10}{\sin(180° - 45°)} = \frac{Q}{\sin(180° - 45°)} = \frac{R}{\sin 90°}$$ $\left(\frac{1}{2}\text{ mark}\right)$

$$\Rightarrow \frac{10}{\sin 45°} = \frac{Q}{\sin 45°} = \frac{R}{\sin 90°}$$ $\left(\frac{1}{2}\text{ mark}\right)$

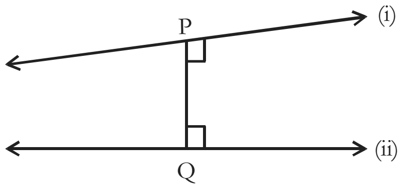
$$\Rightarrow \frac{10}{\left(\frac{1}{\sqrt{2}}\right)} = \frac{Q}{\left(\frac{1}{\sqrt{2}}\right)} = \frac{R}{1}$$ **(1 mark)**

$$\Rightarrow 10 : Q : R = \left(\frac{1}{\sqrt{2}}\right) : \left(\frac{1}{\sqrt{2}}\right) : 1$$

$$\Rightarrow 10 : Q : R = 1 : 1 : \sqrt{2}$$

$\Rightarrow Q = 10$ N, $R = 10\sqrt{2}$ N **(1 mark)**

26.



Given lines are

$$\vec{r} = \left(2\hat{j} - 3\hat{k}\right) + \lambda\left(2\hat{i} - \hat{j}\right) \quad \ldots \text{(i)}$$

$$\vec{r} = \left(4\hat{i} + 3\hat{k}\right) + \mu\left(3\hat{i} + \hat{j} + \hat{k}\right) \quad \ldots \text{(ii)}$$

If PQ be the shortest distance between the lines (i) and (ii) and position vectors of P and Q are

$$2\lambda\hat{i} + (2-\lambda)\hat{j} - 3\hat{k} \quad \ldots \text{(iii)}$$

And $(4+3\mu)\hat{i} + \mu\hat{j} + (3+\mu)\hat{k}$ ... (iv) respectively $\left(\frac{1}{2}\textbf{ mark}\right)$

$\therefore \overrightarrow{PQ}$ = Position vector of Q – Position vector of P

$$= \left[(4+3\mu)\hat{i} + \mu\hat{j} + (3+\mu)\hat{k}\right] - \left[2\lambda\hat{i} + (2-\lambda)\hat{j} - 3\hat{k}\right]$$

$$= (-2\lambda + 3\mu + 4)\hat{i} + (\lambda + \mu - 2)\hat{j} + (\mu + 6)\hat{k}$$ $\left(\frac{1}{2}\textbf{ mark}\right)$

$\overrightarrow{PQ}$ is perpendicular to (i) then $\overrightarrow{PQ}$ is perpendicular to $\left(2\hat{i} - \hat{j}\right)$

$$\therefore \left[(-2\lambda + 3\mu + 4)\hat{i} + (\lambda + \mu - 2)\hat{j} + (\mu + 6)\hat{k}\right] \cdot \left(2\hat{i} - \hat{j}\right) = 0$$

$$\Rightarrow -4\lambda + 6\mu + 8 - \lambda - \mu + 2 = 0$$

$\Rightarrow -5\lambda + 5\mu = -10$

$\Rightarrow \lambda - \mu = 2$ ... (v) **(1 mark)**

Similarly, $\overline{PQ}$ is perpendicular to (ii) then $\overline{PQ}$ is perpendicular to $\left(3\hat{i} + \hat{j} + \hat{k}\right)$

$\therefore \left[(-2\lambda + 3\mu + 4)\hat{i} + (\lambda + \mu - 2)\hat{j} + (\mu + 6)\hat{k}\right] \cdot \left(3\hat{i} + \hat{j} + \hat{k}\right) = 0$

$\Rightarrow -6\lambda + 9\mu + 12 + \lambda + \mu - 2 + \mu + 6 = 0$

$\Rightarrow -5\lambda + 11\mu + 16 = 0$ ... (vi) **(1 mark)**

From (v) and (vi), we get

$\lambda = 1$ and $\mu = -1$ $\left(\frac{1}{2}\textbf{ mark}\right)$

$\therefore \overrightarrow{PQ} = (-2\times 1 + 3\times -1 + 4)\hat{i} + (1 - 1 - 2)\hat{j} + (-1 + 6)\hat{k}$

$= -\hat{i} - 2\hat{j} + 5\hat{k}$

$\therefore \left|\overrightarrow{PQ}\right| = \sqrt{(-1)^2 + (-2)^2 + (5)^2}$

$= \sqrt{1 + 4 + 25}$

$= \sqrt{30}$ **(1 mark)**

Vector equation of PQ is

$\vec{r} = (2\times 1)\hat{i} + (2-1)\hat{j} - 3\hat{k} + \lambda\left(-\hat{i} - 2\hat{j} + 5\hat{k}\right)$ **(1 mark)**

$\Rightarrow \vec{r} = \left(2\hat{i} + \hat{j} - 3\hat{k}\right) + \lambda\left(-\hat{i} - 2\hat{j} + 5\hat{k}\right)$ $\left(\frac{1}{2}\textbf{ mark}\right)$

Section – C

19. Let E_1, E_2 and A be the events such that

E_1 = choosing scooter of Plant – I

E_2 = choosing scooter of Plant – II

A = choosing a scooter of standard quality

$\therefore P(E_1) = \frac{60}{100}$, $P(E_2) = \frac{40}{100}$, $P(A/E_1) = \frac{70}{100}$ and $P(A/E_2) = \frac{80}{100}$

(1 mark)

Probability of choosing a scooter of standard quality from Plant-II is

$$P(E_2/A) = \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}$$ [Using Baye's theorem]

(1 mark)

$$= \frac{\frac{40}{100}\times\frac{80}{100}}{\frac{60}{100}\times\frac{70}{100}+\frac{40}{100}\times\frac{80}{100}}$$

$$= \frac{32}{42+32}$$

$$= \frac{16}{37}$$ **(1 mark)**

20. Let p be the probability of getting head in a single throw.

Therefore, $p = \frac{1}{2}$ and then, $q = \frac{1}{2}$ **$\left(\frac{1}{2}\text{ mark}\right)$**

Let X represents the number of heads in 6 throws.

$$\therefore P(X = r) = {}^6C_r\left(\frac{1}{2}\right)^{6-r}\left(\frac{1}{2}\right)^r = {}^6C_r\left(\frac{1}{2}\right)^6, \text{ where } r = 0, 1, 2, \ldots, 6. \quad \ldots(i)$$

(1 mark)

Now, probability of less than 3 heads

$$= P(X < 3) = P(r = 0) + P(r = 1) + P(r = 2)$$ **$\left(\frac{1}{2}\text{ mark}\right)$**

$$= {}^6C_0\left(\frac{1}{2}\right)^6 + {}^6C_1\left(\frac{1}{2}\right)^6 + {}^6C_2\left(\frac{1}{2}\right)^6$$ **$\left(\frac{1}{2}\text{ mark}\right)$**

$$= \left(\frac{1}{2}\right)^6\left[{}^6C_0 + {}^6C_1 + {}^6C_2\right]$$

$= \frac{1}{64}[1+6+15] = \frac{22}{64} = \frac{11}{32}$ **$\left(\frac{1}{2}\text{ mark}\right)$**

Or

Let p be the probability that a bulb is defective.

$\therefore p = \frac{4}{100}$ and n = 100

$\therefore$ Mean $= m = np = 100 \times \frac{4}{100} = 4$ **(1 mark)**

Let X denote the number of defective bulbs in the sample of 100 bulbs.

$\therefore P(X = r) = \frac{e^{-m} \cdot m^r}{r!} = \frac{e^{-4} \cdot 4^r}{r!}$ **(1 mark)**

P(one defective bulb) = P(X = 1)

$= \frac{e^{-4} \cdot 4^1}{1!}$

$= 4 \times e^{-4}$

$= 4 \times 0.0183$

$= 0.0732$ **(1 mark)**

21. Assume x toys of type P and y toys of type Q be produced per day.

$\therefore$ Profit Z = 3x + 5y **$\left(\frac{1}{2}\text{ mark}\right)$**

Total time per day consumed to prepare x and y toys of type P and Q is x + 2y, which should be less than 200.

$\therefore x + 2y \le 200$

Since plastic is available to produce 150 toys only.

$\therefore x + y \le 150$

Now fancy dress is available for 60 toys per day only.

$\therefore y \le 60$

Hence, linear programming problem is as follows:

Maximum Z = 3x + 5y **(1 mark)**

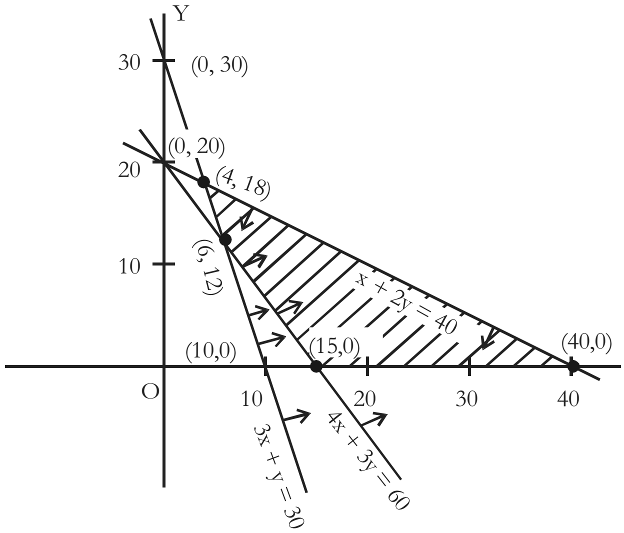
Such that $x + 2y \le 200$ **$\left(\frac{1}{2}\text{ mark}\right)$**

$x + y \leq 150$ $\left(\frac{1}{2}\textbf{ mark}\right)$

$y \leq 60$ $\left(\frac{1}{2}\textbf{ mark}\right)$

and $x \geq 0,\ y \geq 0$

22. First consider the constraints as equalities.
$x + 2y = 40$ … (i)
$3x + y = 30$ … (ii)
$4x + 3y = 60$ … (iii)
Here draw the lines in two-dimensional plane.



(1 mark)

Substitute $y = 0$ in (i), we get $x = 40$ and $x = 0 \Rightarrow y = 20$
Similarly in equation (ii) and (iii), we get
In (ii), $y = 0 \Rightarrow x = 10$ and $x = 0 \Rightarrow y = 30$,
In (iii), $y = 0 \Rightarrow x = 15$ and $x = 0 \Rightarrow y = 20$
Solving (ii) and (iii), we get the point of intersection (6, 12).

Solving (i) and (ii), we get the point of intersection (4, 18).
From the permissible region, the point of intersections are (40, 0), (15, 0), (6, 12) and (4, 18).

For (40, 0), Z = 20 × 40 + 10 × 0 = 800 $\left(\frac{1}{2}\text{ mark}\right)$

For (15, 0), Z = 20 × 15 + 10 × 0 = 300

For (6, 12), Z = 20 × 6 + 10 × 12 = 240 $\left(\frac{1}{2}\text{ mark}\right)$

For (4, 18), Z = 20 × 4 + 10 × 18 = 80 + 180 = 260 $\left(\frac{1}{2}\text{ mark}\right)$

Hence, Z is minimum for x = 6 and y = 12. $\left(\frac{1}{2}\text{ mark}\right)$

23. ∵ X and Y share profits and losses in the ratio 4 : 3.

∴ X's share of premium = Rs. $\left(\frac{4}{7}\times 91000\right)$ = Rs. 52,000 $\left(\frac{1}{2}\text{ mark}\right)$

Y's share of premium = Rs. $\left(\frac{3}{7}\times 91000\right)$ = Rs. 39,000 $\left(\frac{1}{2}\text{ mark}\right)$

Let p be the total profit

Z's share in the profit = $\frac{p}{4}$ $\left(\frac{1}{2}\text{ mark}\right)$

Combined share of X and Y in the profit $= p - \frac{p}{4} = \frac{3p}{4}$ **(1 mark)**

∵ It is to be divided in the profit sharing ratio

∴ X's share = $\frac{3p}{4}\times\frac{4}{7} = \frac{3p}{7}$ $\left(\frac{1}{2}\text{ mark}\right)$

Y's share = $\frac{3p}{4}\times\frac{3}{7} = \frac{9p}{28}$ $\left(\frac{1}{2}\text{ mark}\right)$

Z's share = $\frac{p}{4}$

$\therefore$ New profit sharing ratio of X, Y and Z

$= \frac{3p}{7} : \frac{9p}{28} : \frac{p}{4}$ **$\left(\frac{1}{2}\text{ mark}\right)$**

= 12 : 9 : 7

Or

The equivalent capital of X $= \text{Rs. } 6000 \times 12 = \text{Rs. } 72,000$

The equivalent capital of Y $= \text{Rs. } (3000 \times 6 + 4000 \times 6) = \text{Rs. } 42,000$

The equivalent capital of Z $= \text{Rs. } (2000 \times 4 + 5000 \times 8) = \text{Rs. } 48,000$

$\left(1\frac{1}{2}\text{ marks}\right)$

X: Y: Z = 72,000: 42,000: 48,000 = 12: 7: 8

Sum of the ratios = 12 + 7 + 8 = 27 **(1 mark)**

Annual profit = Rs. 4, 320

X's share $= \text{Rs. } \frac{4320 \times 12}{27} = \text{Rs. } 1,920$

Y's share $= \text{Rs. } \frac{4320 \times 7}{27} = \text{Rs. } 1,120$

Z's share $= \text{Rs. } \frac{4320 \times 8}{27} = \text{Rs. } 1,280$ **$\left(1\frac{1}{2}\text{ marks}\right)$**

24. Installment = Rs. 1,200 per annum, time = 2 years and rate = 6% per annum.

$\therefore a = 1200,\ n = 2,\ r = 6 \text{ and } i = \frac{r}{100} = \frac{6}{100} = 0.06$ **(1 mark)**

Let P be the present value of the annuity due

$\therefore P = \frac{a(1+i)}{i}\left[1 - (1+i)^{-n}\right]$ **(1 mark)**

$= \frac{1200(1+0.06)}{0.06}\left[1 - (1+0.06)^{-2}\right]$ **(1 mark)**

$= \frac{120000}{6}\left[1.06 - (1.06)^{-1}\right]$

$= 20000[1.06 - 0.943]$ $\left[\text{Given } (1.06)^{-1} = 0.943\right]$

$= 20000 \times 0.117$ $=$ Rs. 2,340 **(1 mark)**

25. Given $C = 100x - 10x^2 + \frac{x^3}{3}$ … (i)

Differentiating (i) with respect to x, we get

$$\frac{dC}{dx} = 100 - 20x + x^2$$

$\Rightarrow MC = 100 - 20x + x^2$ … (ii) $\left[\because \text{Marginal Cost} = \frac{dC}{dx}\right]$ **(1 mark)**

Differentiating (ii) with respect to x, we get

$\frac{d}{dx}(MC) = 0 - 20 + 2x$ … (iii)

MC is minimum if $\frac{d}{dx}(MC) = 0$

$\Rightarrow -20 + 2x = 0$

$\Rightarrow x = 10$ $\left(\mathbf{1\frac{1}{2}\ marks}\right)$

Differentiating (iii) with respect to x, we get

$\frac{d^2}{dx^2}(MC) = 2 > 0$ [For x = 10]

$\therefore$ MC is minimum when x = 10, i.e. when the output is of 10 units.

$\left(\mathbf{1\frac{1}{2}\ marks}\right)$

26. Amount of the bill = Rs. 30,000
Date on which the bill was drawn = March 4, 2002
Due date = January 4, 2002 [After 10 months]
Legal due date = January 4, 2002 + 3 days of grace = January 7, 2002

$\left(\mathbf{\frac{1}{2}\ mark}\right)$

Date on which the bill was discounted = August 14, 2002 [Given]
Time between date of discounting and date of maturity of bill

August	September	October	November	December	January
18	+30	+31	+30	+31	+6

= 146 days $= \frac{2}{5}$ years **(1 mark)**

Banker's discount $= \text{Rs.}\left(\frac{30000 \times \frac{2}{5} \times 5}{100}\right) = \text{Rs.}\,600$ [Since, rate = 5%]

(1 mark)

Amount paid by the banker = Rs. (30, 000 – 600) = Rs. 29, 400 $\left(\frac{1}{2}\ \textbf{mark}\right)$

Present worth of the bill on the day it is discounted

$$= \text{Rs.}\left(\frac{30000 \times 100}{100 + \frac{2}{5} \times 5}\right) = \text{Rs.}\frac{30000 \times 100}{102} = \text{Rs.}\,29411.76$$ **(1 mark)**

True discount = Rs. (30,000.00 – 29411.76) = Rs. 588.24 **(1 mark)**

Therefore, banker's gain = Banker's discount – True discount

= Rs. (600 – 588.24) = Rs. 11.76 **(1 mark)**

Biology

Section – A

1. Abscisic acid prevents excessive loss of water hence it is called stress hormone. **(1 mark)**

2. Malpighian tubules **(1 mark)**

3. Acrosome of spermatozoon helps in penetration of the spermatozoon into the ovum, hence, if it is not functioning properly there would not be any fertilization. **(1 mark)**

4. Biodiversity is useful as:

a) Source of new crops $\left(\frac{1}{2}\text{ mark}\right)$

b) Source of material to improve variety of crops $\left(\frac{1}{2}\text{ mark}\right)$

c) Source of new natural biodegradable pesticides $\left(\frac{1}{2}\text{ mark}\right)$

(write any two of the above points)

5. Ciprofloxacin **(1 mark)**

Section – B

6. Imbibition takes place in following two conditions:

a) Water potential gradient between the surface of the adsorbant and the liquid imbibed. **(1 mark)**

b) Affinity between the adsorbant and the imbibed liquid. **(1 mark)**

7. Ectotherms are animals in which the body temperature tends to match with the environmental temperature in which they live. **(1 mark)**
Endotherms are those animals, which regulate their body temperatures by physiological means and maintain more or less constant internal temperature. **(1 mark)**

8. Insectivorous plants generally grow in water-logged, swampy soil which are deficient in nitrogen. They have poorly developed roots and hence, have to depend upon the nitrogen from animal body. **(1 mark)**
They also have chlorophyll to prepare their own food.
Examples: Pitcher plant *(Nepenthes)*, sundew *(Drosera)*, bladder wort *(Utricularia)*, butter wort *(Pinguicula)*, venus fly-trap *(Dionaea)*, etc.
(1 mark)

9. The plant is deficient of:

a) Nitrogen $\left(\frac{1}{2}\text{ mark}\right)$

b) Phosphorus $\left(\frac{1}{2}\text{ mark}\right)$

c) Magnesium $\left(\frac{1}{2}\text{ mark}\right)$

d) Boron $\left(\frac{1}{2}\text{ mark}\right)$

10. The functions of cerebrospinal fluid are as follows:

a) Provide shock-absorbing medium to brain and spinal cord. $\left(\frac{1}{2}\text{ mark}\right)$

b) Give buoyancy to brain. $\left(\frac{1}{2}\text{ mark}\right)$

c) Help in excretion of wastes products. $\left(\frac{1}{2}\text{ mark}\right)$

d) Act as endocrine medium to transport hormones to other areas of brain.
$\left(\frac{1}{2}\text{ mark}\right)$

11. The stages of spermatogenesis are as follows:
a) Spermatocytogenesis: Spermatogonia divide to form spermatocytes
b) Meiosis: Spermatocytes undergo reduction division
c) Spermiogenesis: Spermatids differentiate into spermatozoa.
Explanation of all three stages **(1 mark)**

12. The flowers that are pollinated by bats possess following characteristics:
a) Bloom after sunset, and are large mostly white with strong scent. **(1 mark)**
b) Have more nectar, prominent stamens that produce more pollen grains. **(1 mark)**

13. Habitat is a specific place or locality where an organism lives. **(1 mark)**
A microhabitat is an area within a given habitat, having some special features suitable for a particular type of organisms than others. **(1 mark)**

14. Diphtheria, Pertussis and Tetanus: DPT $\left(\frac{1}{2}\textbf{ mark}\right)$

Hepatitis: Hepatitis B $\left(\frac{1}{2}\textbf{ mark}\right)$

Polio: Polio $\left(\frac{1}{2}\textbf{ mark}\right)$

Tuberculosis: BCG $\left(\frac{1}{2}\textbf{ mark}\right)$

15. Sonography is a method of diagnostic medical imaging using a high frequency sound wave. **(1 mark)**
It works on the principle of sonar. **(1 mark)**

Or

Recombinant DNA technology allows production of antigenic polypeptides of pathogens in transgenic organisms. **(1 mark)**
Example: Hepatitis B **(1 mark)**

Section – C

16. The factors affecting photosynthesis are:

a) Light intensity and quality $\left(\frac{1}{2}\text{ mark}\right)$

b) Availability of carbon dioxide $\left(\frac{1}{2}\text{ mark}\right)$

c) Availability of water $\left(\frac{1}{2}\text{ mark}\right)$

d) Temperature $\left(\frac{1}{2}\text{ mark}\right)$

e) Age of leaf, its anatomy and chlorophyll content also affect photosynthesis. $\left(\frac{1}{2}\text{ mark}\right)$

f) The number of stomata, their opening and closing, venation of leaf, volume of intercellular spaces influence the rate of photosynthesis. $\left(\frac{1}{2}\text{ mark}\right)$

17. Photosynthesis is crucial because:

a) All green plants prepare organic food material for all living organisms on earth by the process of photosynthesis. **(1 mark)**

b) It is the only naturally occurring process that evolves oxygen. It maintains atmospheric oxygen level, which is continuously utilized by plants and animals during respiration. **(1 mark)**

c) If photosynthesis would not take place, there would not be any food for any living organism on earth thus, causing death of all life on earth. **(1 mark)**

18. Monoglycerides, fatty acids and glycerol are first incorporated into water-soluble droplets micelles as a combination of fatty acids, monoacylglycerols and bile salts. **(1 mark)**

Micelles are absorbed by facilitated diffusion through brush border membrane. **(1 mark)**

The triglycerides combine with phospholipids and cholesterol, which are released into lymph in the form of protein-coated lobules called chylomicrons.

These are then transported through blood to all parts of the body. **(1 mark)**

19. Electrocardiograph will enable us to detect heart rate. **(1 mark)**
When the nerve bundles are inflamed or infected, the distance between the first spike, denoting atrial contraction and the second spike denoting ventricular contraction is greater than normal. **(2 mark)**

20.

Arteries	**Veins**
a) Are blood vessels that carry blood away from the heart	a) Are blood vessels that carry blood to the heart
b) Are thick-walled	b) Are thin-walled
c) Made up of three layer tunica externa, tunica media and tunica intima	c) Also made up of three layers: tunica externa, tunica media and tunica intima; but tunica media is much thinner than it is in arteries
d) Do not have valves	d) Have valves
e) More elastic than veins	e) Less elastic than arteries
f) Carry oxygenated blood, except for the pulmonary artery	f) Carry deoxygenated blood, except for the pulmonary vein

(Any three points) **(1 mark for each point)**

21. The natural ability of an organism to restore the missing parts of the body is called regeneration. **(1 mark)**
If a *Hydra* is cut into two parts, these parts can grow into new individuals. These parts retain their original polarity with oral ends developing tentacles and aboral ends developing basal discs. Small fragments of sponges and also flatworms can grow in complete individuals.
Arthropods such as crabs, lobsters, spiders and certain insects can regenerate their lost legs. The starfishes cut off their arms when injured and then regenerate new ones. Some fish can regenerate their fins. The amphibians such as newts, salamanders and axolotl larvae can regenerate their lost or severed arms or legs. Reptiles such as lizards can regenerate their lost tail. Mammals can regenerate their liver.
(Only four examples are expected)

(Four examples carry) $\left(\frac{1}{2}\ \textbf{mark each}\right)$

22. Some of the man-made methods of vegetative propagation are:
Cutting: small piece of a plant organ such as stem, root or leaf is used for propagation.

$\left(\frac{1}{2}\text{ mark}\right)$

Layering: includes induction of roots on stem branches before they are detached from the parent plant. Layering is of two types:
a) Mound layering
b) Air layering or gootee **(1 mark)**

Grafting: Parts of two plants, generally stem or branches, are joined in such a way that they grow as one plant. The plant, which is rooted is called the stock and the twig of another plant joint on it is called the scion. Types of grafting
a) Tongue or whip grafting
c) Wedge grafting
c) Crown grafting
d) Side grafting
e) Bud-grafting **(1 mark)**

Plant tissue culture or micropropagation: Propagation of plant is done by means of tissue culture.

$\left(\frac{1}{2}\text{ mark}\right)$

23. A food chain can be defined as the sequence of eating and being eaten, with the resultant transfer of energy. **(1 mark)**

When the primary consumer (herbivore) consumes producers (plants), only 10 per cent energy is transferred. Similarly, only 10 per cent energy is transferred from primary consumer to the secondary consumer (carnivore). For this, Lindemann in 1942 formulated a concept called as 10 per cent law. According to this law, only 10 per cent of the energy is available for transfer to the next trophic level. **(2 mark)**

Or

The three kinds of freshwater wetlands are:
Marshes which are defined as wetlands frequently inundated with water. These are characterized by emergent soft-stemmed vegetation adapted to saturated soil conditions. **(1 mark)**

Swamps are any wetlands dominated by woody plants. They are characterized by saturated soils during the growing season, and standing water during certain times of the year. **(1 mark)**

Bogs are characterized by deposits of spongy peat, acidic waters, and a floor covered by a thick carpet of sphagnum moss. **(1 mark)**

24. The heart's electrical activity is charted on an ECG. The P-wave represents signals passing from sino-atrial node to atrioventricular node. **(1 mark)**
There is a deep in the chart denoted as Q which records signals passing along the bundle of His. **(1 mark)**
R and S points mark their passage along the bundle branches and Purkinjes fibres. T denotes the relaxation during diastole. **(1 mark)**

25. Allopolyploidy is the polyploidy which is the resultant of chromosome doubling following hybridization of two distinct species. **(1 mark)**
A cross between cabbage, *Brassica oleracea* and radish, *Raphanus sativus* produces a hybrid which has one set of cabbage and one of radish. **(1 mark)**
Triticale is the first man-made allopolyploid developed by crossing wheat, *Triticum* and rye, *Secale.* **(1 mark)**

Section – D

26. The following table summarizes the gastrointestinal hormones and their functions:

Hormone	Action
Gastrin	a) Stimulates acid secretion and also pepsin secretion. b) Stimulaes bile flow. c) Inhibits water and electrolyte absoption in intestine d) Stimulates contraction of gastroesophageal sphinctor and relaxes sphinctor of oddix, and the ileal-caecal sphinctor.
Secretin	a) Stimulates pancreatic acinii and bile secretion. b) Stimulates bile ducts to secrete water and bicarbonate ions. c) Inhibits gastric secretion of gastrin and peristalsis. d) Stimulates pepsin secretion and insulin secretion.

Cholecystokinin — pancreozymin	a) Stimulates enzyme secretion from pancreatic acinii. b) Stimulates contraction and evacuation of gall bladder. c) Inhibits gastric secretion
Gastric inbitory peptide (GIP)	a) Inhibits HCI and pepsin secretion b) Stimulates insulin secretion
Vasoactive intestinal peptide (VIP)	a) Affects arterial vasodilation and hypotension. b) Increases splanic blood flow. c) Suppresses gastric acid secretion
Somatostatin	a) Inhibits release of CCK,VIP and GIP secretion. b) Decreases intestinal and gall bladder contraction c) Decreases bile flow.
Enteroglucagon	Has tropic effects on enterocytes.
Motilin	Stimulates gastric motor activity
Neurotensin	Inhibits gastric motor activity.

(From the above-mentioned hormones the first five are very important.

(1 mark each for any five hormones.)

27. The main reasons of deforestation are overpopulation of human and livestock, increased demand for timber and fuel wood, expansion of agricultural lands and overgrazing. Also, construction of roads along the mountains, industrialization, mining, quarrying, irrigation, etc. have led to deforestation. Paper industry has led to overfelling of trees. **(3 marks)**

Indiscriminate cutting down of forests have the following adverse effects:

a) Reduction in rainfall in that area causing fewer trees to grow, and ultimately making the area a desert.

$$\left(\frac{1}{2}\textbf{ mark}\right)$$

b) Loss of fertile soil through wind and water erosion. Roots of trees hold soil firmly and do not allow it to erode away.

$\left(\frac{1}{2} \text{ mark}\right)$

c) Frequent floods in the area.

$\left(\frac{1}{2} \text{ mark}\right)$

d) Destruction of natural habitat of many animals.

$\left(\frac{1}{2} \text{ mark}\right)$

Or

27. The effects of air pollution on environment are as follows:

a) Increase of Carbon dioxide in air has started causing 'greenhouse effect'. **(1 mark)**

b) Nitrogen dioxide and oxides of sulphur react with water vapour and come down as acid rains. Acid rains endanger crop plants and vegetables. They destroy marble monuments. Sulphuric acid attacks metal surfaces, such as steel, rail tracks, etc. **(1 mark)**

c) Hydrogen sulphide tarnishes silver and blackens lead-based paints. **(1 mark)**

d) CFCs of aerosol sprays deplete the ozone layer. This permits an entry of harmful ultraviolet radiation on earth. **(1 mark)**
Dust and smoke do not allow a clear view of nature's beauty. Foulodours emitted by industries, automobiles, dirty drains, garbage, etc. cause nuisance. **(1 mark)**

28. The diseases in animals can be controlled by taking following measures:

a) Isolating infected animals from the healthy ones. $\left(\frac{1}{2} \text{ mark}\right)$

b) Disposing carcass properly. $\left(\frac{1}{2} \text{ mark}\right)$

c) Regular cleaning of animal shelters. $\left(\frac{1}{2} \text{ mark}\right)$

d) Giving vaccines regularly. $\left(\frac{1}{2} \text{ mark}\right)$

e) Transferring animals to a pastures not used by diseased animals.

$\left(\frac{1}{2}\text{ mark}\right)$

f) Informing the veterinary authorities immediately if any case of infectious disease detected.

$\left(\frac{1}{2}\text{ mark}\right)$

Anthrax is an infectious disease caused by spore-forming bacterium *Bacillus anthracis*. It occurs in cattle, sheep, goats, camels, antelopes and other herbivores. It can occur in humans too. **(1 mark)**

There are cases of sudden deaths when animals contract anthrax. Animal suffers from fever, lack of rumination, excitement followed by depression, difficulty in breathing, uncoordinated movements, convulsions, bloody discharges from natural body openings. **(1 mark)**